

Statistical Analysis of Long-Term Operating Structures for Various Purposes Based on the Theory of Probabilistic Outliers

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Abstract. A probabilistic approach to assessing the reliability of long-term operating mechanical-engineering and building structures based on the theory of probabilistic outliers is considered. The aim of the study was to develop a methodology for determining the probability of failure-free operation of structural elements while accounting for the statistical variability of loads and load-bearing capacity during long-term service. Unlike traditional limit-state calculations, the proposed approach takes into account the statistical parameters of the load-resistance system and actual data from technical inspections of structures. Analytical relationships were obtained for determining the probability of failure-free operation, the safety characteristic, and the strength reserve factor. A probabilistic analysis of a steel beam and a reinforced concrete structure was performed using statistical data from technical inspections. The results showed that the proposed method makes it possible to quantitatively assess operational reliability, predict residual service life, and substantiate engineering decisions on further operation, strengthening, and reconstruction of building structures.

Keywords: reliability of mechanical-engineering and building structures, probability of failure-free operation, theory of probabilistic outliers, statistical analysis, residual service life, operational reliability.

Introduction

The loading processes of mechanical-engineering and building structures, as well as material characteristics and the load-bearing capacity levels of buildings and facilities, are random in nature. The variability of loads, mechanical properties of materials, and geometric parameters of structural elements necessitates the use of probabilistic methods for reliability analysis.

The widely used method for calculating structures for various purposes by limit states has, in fact, a probabilistic interpretation, since the interaction between loads and element resistance is considered in terms of the possibility of reaching limiting states. However, the regulatory approach in most cases provides only an upper estimate of reliability, because it uses normative safety factors and design values of loads and resistances. At the same time, one of the main problems remains the establishment of statistical relationships between random parameters that determine the stress-strain state of a structure [1].

Under real operating conditions, loads, material properties, and geometric parameters of structures change over time and are characterized by a random nature. In this regard, the technical condition of a building object should be considered as a set of random processes describing changes in the parameters of the load-resistance system. The factor parameters of the system may be treated as random variables determined from statistical observations and technical inspections [2].

Estimates of random parameters must satisfy the requirements of consistency, unbiasedness, and efficiency. A consistent estimate is one that tends toward the true value of a parameter as the number of observations increases. An unbiased estimate is one whose mathematical expectation coincides with the true value of the parameter. An efficient estimate is one that has the minimum variance among all possible estimates.

When studying the reliability of mechanical-engineering and building structures, probability theory and mathematical statistics are used to analyze the variability of the parameters of the load-resistance system. One effective approach is the theory of probabilistic outliers, in which structural failure is considered as the event of a random load process exceeding a certain critical level [3].

The theory of probabilistic outliers is one of the areas of probabilistic reliability analysis of mechanical-engineering structures operating under random operational and dynamic effects. The basic idea of this approach is to consider the loading process as a random process in which structural failure is associated with stresses or strains exceeding a dangerous limiting level [4-6].

In mechanical-engineering structures, the random nature of loads is caused by variable operating modes, vibrations, impact effects, wear of elements, and variability in material properties. In this regard, the parameters of the stress-strain state are considered as random functions of time, while structural reliability is determined by the probability that the specified limiting state will not be exceeded [7-9].

The practical value of the theory of probabilistic outliers lies in its ability to assess the probability of failure, predict residual service life, and substantiate engineering decisions regarding further operation, repair, and strengthening of mechanical-engineering and building structures [10-11].

This approach has several advantages [12-14]:

- it directly links structural reliability with the frequency of exceedance of a dangerous load level;
- it accounts for the statistical variability of the load-resistance system parameters;
- it can be used to analyze long-term operating structures based on inspection data.

Existing methods for calculating the reliability of various structures are usually focused on design values of loads and resistances and rarely account for the actual statistical variability of parameters that arises during operation [15-17].

Therefore, it is necessary to develop methods that combine:

- probabilistic reliability theory;
- statistical data from technical inspections of structures;
- stochastic modeling of load-resistance interaction.

The main reliability indicator for building structures is the probability of failure-free operation. It characterizes the probability that a structure will not reach a limiting state within a specified operating period. This study uses a methodology for determining the probability of failure-free operation based on the condition that the load does not exceed a dangerous level.

The scientific novelty of the study is as follows:

- a methodology for the probabilistic assessment of structural reliability based on the number of load crossings of a dangerous level has been developed;
- an approach has been proposed for accounting for statistical inspection data when assessing the reliability of operating structures;
- analytical relationships have been obtained between the safety characteristic, the strength reserve factor, and the statistical parameters of loads and resistance.

The practical usefulness of the study lies in the development of a probabilistic methodology for assessing the operational reliability of building structures while accounting for the statistical variability of loads and load-bearing capacity. The proposed approach makes it possible to assess the probability of failure-free operation, predict residual service life, and substantiate decisions on strengthening and further operation of facilities for various purposes [18-20].

1. Materials and Methods

The main criterion for assessing the reliability of building structures and facilities is the probability of failure-free operation (PFFO). Its value is established on the basis of a number of conditions related to the limitation of the ultimate load, the level of load-bearing capacity, the degree of physical wear, and other operational factors. This paper considers a methodology for calculating the probability of failure-free operation under the condition that the load does not exceed a level dangerous for operation [5, p. 98-105].

It has been established that in this case the realization of the corresponding random process is close to the Poisson distribution law, and the crossing of the dangerous level of external load can be described as a nonstationary Poisson flow. Then the lower estimate of the probability of failure-free operation has the following form (PFFO) [1, p. 68; 2, p. 102]:

$$P_{h.o.}(T) = 1 - b, \quad (1)$$

where T is the operating time of the system, service life, or operating period; b is the average number of load crossings of the dangerous level over time T .

Let the studied factors be denoted as follows: acting load (stress); load-bearing capacity; and the resulting strength reserve parameter.

After transforming the probabilities of crossings of two random processes, the expression for the function b can be written as:

$$\begin{aligned} b &= -T \int_{-\infty}^0 f[\dot{S}_{\Sigma}(t), S_{\Sigma}(t) = 0] \cdot \dot{S}_{\Sigma}(t) d\dot{S}_{\Sigma}(t) = \\ &= \left\{ T \sigma_s \cdot \exp \left[\left(-\langle S_{\Sigma} \rangle^2 : 2 : \sigma_s^2 \right) \right] : (2\pi\sigma_s) \right\}, \end{aligned} \quad (2)$$

where the terms denote the mathematical expectations and standard deviations of the corresponding random processes.

For an ergodic stationary process with a normal distribution law, we obtain [21]:

$$b = T \cdot T_e^{-1} \exp \left[- (S_0 - \langle S \rangle)^2 : (2\sigma_s^2) \right], \quad (3)$$

where the effective period of the random process is numerically equal to the average value of the loading-process period; S_0 is the dangerous loading level; and the remaining term denotes the average loading level.

The correlation function and mathematical expectation required to determine the average number and average duration of outliers for a normal stationary random process describing structural operation have the following form [22, 23]:

$$K_S(t) = D_h \rho_h(\tau) + D_e \rho_e(\tau) + (D_h D_e)^{1/2} \rho_{h,e}(\tau), \quad (4)$$

where the subscripts 'l' and 'h' correspond to the low-frequency and high-frequency components of the object loading process.

When the sign of the load is not significant and the load-bearing capacity is a random variable distributed according to a truncated normal law, the following expression can be written instead of equations (2) and (3):

$$b = C_0 \cdot \sigma_s \cdot T \cdot \left[2\pi(\sigma_s^2 + \sigma_0^2)^{1/2} \right]^{-1} \exp \left[- (\langle S_0 \rangle - \langle S \rangle)^2 : 2 : (\sigma_s^2 + \sigma_0^2) \right], \quad (5)$$

where C_1 and C_2 are the normalizing coefficients of the truncated normal distribution for loads and load-bearing capacity, respectively. They are calculated as follows [24]:

$$C_1 = \left[0, 5 \left\{ \Phi \left[(S_{\max} - \langle S \rangle) : (\sigma_s \cdot 2^{1/2}) \right] - \Phi \left[(S_{\min} - \langle S \rangle) : (\sigma_s \cdot 2^{1/2}) \right] \right\} \right], \quad (6)$$

$$C_2 = \left[0, 5 \left\{ \Phi \left[(S_{0,\max} - \langle S_0 \rangle) : (\sigma_0 \cdot 2^{1/2}) \right] - \Phi \left[(S_{0,\min} - \langle S_0 \rangle) : (\sigma_0 \cdot 2^{1/2}) \right] \right\} \right], \quad (7)$$

where Φ is the tabulated Laplace function, $S_{\max}$ and $S_{\min}$ are the maximum and minimum load values, and $S_{0,\max}$ and $S_{0,\min}$ are the corresponding values of the dangerous loading level. In practical calculations, it may be assumed that $C_1 = C_2 = C_0 = 1.0$.

In the present study, the normal distributions of the load parameter S and the limiting load-bearing parameter S_0 are truncated over their physically admissible domains, i.e., $S_{\min} = 0$, $S_{\max} \rightarrow \infty$, $S_{0,\min} = 0$ and $S_{0,\max} \rightarrow \infty$. The lower truncation limit eliminates physically impossible negative values, whereas no artificial upper truncation (e.g., at 3σ) is introduced. These limits are used in determining the normalization coefficients C_1 and C_2 in eqs. (6) and (7). For the statistical parameters considered in this study, the probability mass below zero is negligible; therefore, the normalization coefficients differ only slightly from unity and the approximation $C_1 = C_2 = C_0 \approx 1$ is adopted in the subsequent derivation of eqs. (8)–(10).

Then, instead of equation (5), we obtain:

$$b = T \cdot b_c \left(1 + \sigma_0^2 : \sigma_s^2 \right)^{-1/2} \exp \left[- (\langle S_0 \rangle - \langle S \rangle)^2 : 2 : (\sigma_s^2 + \sigma_0^2) \right], \quad (8)$$

where the parameter denotes the number of outliers per unit time, determined by expert assessment.

Let us further consider the application of the probabilistic calculation method to the case of allowable stresses. Based on expression (8), we obtain the following relationship:

$$(\langle S_0 \rangle - \langle S \rangle) (\sigma_s^2 + \sigma_0^2)^{-\frac{1}{2}} = \gamma, \quad (9)$$

where gamma is the safety characteristic, by analogy with [4, p. 146].

This characteristic can be determined using the expression

$$\gamma = \left[\left[-2 \ln \left\{ -l_p [p] (Tb_c Q)^{-1} \right\} \right] \right]^{1/2}, \quad (10)$$

where the parameter denotes the normative probability of failure-free operation (PFFO).

The parameter Q is determined by the expression [24, p.4]:

$$Q = \left(1 + \sigma_0^2 : \sigma_s^2 \right)^{-1/2} = \varphi \left(1 + \varphi^2 \right)^{-1/2}, \quad (11)$$

where the coefficient determines the ratio related to the standard deviation (SD).

Then, using (9), the required strength reserve factor K can be determined:

$$K = \langle S_0 \rangle : \langle S \rangle = \varphi^{-1} \left[\varphi + \gamma \cdot \nu_s \left(1 + \varphi^2 \right)^{1/2} \right], \quad (12)$$

where

$$\nu_s = \sigma_s : \langle S \rangle \quad (13)$$

is the coefficient of variation of stress.

The value K determined by expression (12) represents the ratio of the average value of load-bearing capacity to the load value.

The condition of probabilistic strength is written as follows [3, p. 87]:

$$\langle S_{no} \rangle : (R) \geq \langle S_H \rangle, \quad (14)$$

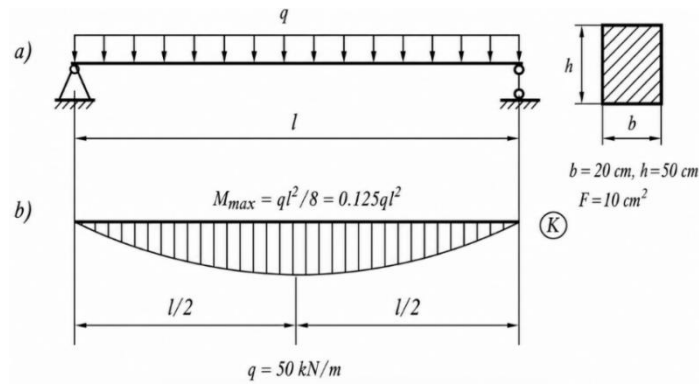
where $\langle S_H \rangle$ – the terms denote normal stress in the structure; $\langle R \rangle$ – the design resistance of the material.

The proposed methodology makes it possible to assess the reliability of building structures from the standpoint of probability theory. The main advantage of this approach is that it accounts for the fact that loads, material strength, and external effects continuously change over time, and these changes are random. Unlike rigid classical calculations, this model makes it possible to calculate the real probability that a structure will operate without failure. It links strength reserve, safety indicators, and load-resistance statistics, which is critically important for objects that have been in service for many years.

This mathematical apparatus can be confidently used to verify the technical condition of buildings based on monitoring results and to accurately predict their residual service life. The probabilistic outlier method captures the real picture: element wear, fluctuations in the stress-strain state, and actual operating conditions of the object. As a result, calculations become much more accurate, allowing engineers to make balanced and justified decisions on whether structures should be strengthened, whether reconstruction is required, or whether the building can continue to be used.

3. Results and Discussion

To illustrate the theoretical provisions of the methodology proposed above, let us consider an example of statistical analysis of the load-bearing capacity of a beam, the design scheme of which is shown in Figure 1 [25-27].



a) initial scheme; b) bending moment diagram.

Fig. 1 – Beam design scheme

Let us introduce the following designations: S_H – normal stress in the structure; $S_{p.c} = S_{h.F} = S_h(b \cdot h)$ – load-bearing capacity of the structure under tension-compression; $S_{iz} = S_h \cdot w = S_h(b \cdot h^3/6)$ – and the same parameter under transverse bending.

For the beam under consideration, the mathematical expectation of the normal stress from the maximum bending moment M_{max} (Figure 1) is assumed, $\langle S_h \rangle = 1 \cdot 10^3 \text{ kgs/cm}^2 = 10^4 \text{ kPa}$ and the yield strength of the steel material is taken as the specified value $\langle \sigma_T \rangle = \langle \sigma_{ho} \rangle = 2,7 \cdot 10^3 \text{ kgs/cm}^2 = 2,7 \cdot 10^4 \text{ kPa}$.

For further calculation, the following initial data are adopted: the average number of outliers beyond the average level of normal stress $\langle S_h \rangle - b_c = 5 \cdot 10^{-2} \text{ sec}^{-1}$; the operating time of the structure under consideration – $T = 7 \text{ years} = 2 \cdot 10^8 \text{ sec}$; the normative (specified) probability of failure-free operation (PFFO) – $([p] = 0,99)$; $(v_s = \sigma_{ho} / \langle S_h \rangle = 0,3)$ – the coefficient of variation of normal stresses (conditionally assigned value).

Then the standard deviation (SD) σ_{Sh} of normal stresses is determined through the coefficient of variation v_s :

$$\sigma_{Sh} = v_s \cdot \langle S_h \rangle = 0,3 \cdot 1 \cdot 10^3 \text{ kgs/cm}^2 = 0,3 \cdot 10^4 \text{ kPa}.$$

The standard deviation of the dangerous-section limiting stress ($\sigma_{h.o}$) is determined through the adopted value ($\varphi = \sigma_{Sh} / \sigma_{ho} = 1,66$) as follows:

$$\sigma_{ho} = \frac{\sigma_{Sh}}{\varphi} = \frac{0,3 \cdot 10^3}{1,66} = 0,1807 \cdot 10^3 \left(\frac{\text{kgs}}{\text{cm}^2} \right) = 1,807 \cdot 10^3 \text{ kPa}.$$

We now proceed to solve the probabilistic analysis problem for the structure shown in Figure 1. Using expressions (10) and (11), the safety characteristic is determined [28]:

$$\gamma = -2 \ln \left[\frac{-\ln 0,99 (1 + 1,66^2)^{1/2}}{1,66 \cdot (5 \cdot 10^{-2}) (2 \cdot 10^8)} \right]^{1/2} = 6,41; \quad \boxed{\gamma} \quad (15)$$

$$Q = 1,66 + (1 + 1,66^2)^{-1/2}.$$

Using formula (12), the strength reserve factor of the structure is calculated:

$$K = \frac{1,66 + 6,41 \cdot 0,3(1 + 1,66^2)^{1/2}}{1,66} = 3,24. \quad (16)$$

Using formula (13), the strength condition of the structure is checked with allowance for value (16):

$$\frac{\langle S_{ho} \rangle}{R} \geq \langle S_n \rangle, \quad \text{or} \quad \left(\frac{2,7 \cdot 10^3}{3,24} = 0,835 \cdot 10^3 \right) < 1 \cdot 10^3 \left(\frac{\text{kgf}}{\text{cm}^2} \right). \quad (17)$$

The strength check (17) shows that, with the initial cross-section, the reliability condition is not satisfied. This means that further operation requires an increase in the cross-sectional dimensions. Thus, the probabilistic calculation not only records the fact of insufficient strength, but also allows the required level of structural strengthening to be quantitatively assessed.

The required section modulus of the cross-section is determined by the expression:

$$W_{mp} = \langle S \rangle R : \langle S_{ho} \rangle, \quad (18)$$

where: $W = bh^2 / 6$ ($b \times h = 2 \times 5\text{cm}$); $W = 0,167 \cdot Fh$; $F = 10\text{cm}^2$; $b = 2\text{cm}$; $h = 5\text{cm}$.

Let us determine the mathematical expectation of the load-bearing capacity, assuming that the structure operates under transverse bending (Figure 1):

$$\langle S \rangle = \langle S_n \rangle \cdot w = 1 \cdot 10^3 \cdot 0,167 \cdot 10 \cdot 5 = 8,35 \cdot 10^3 \text{ (kgf} \cdot \text{m)}.$$

Let us determine the required value of the cross-sectional section modulus with allowance for (16) and (17):

$$W_{mp} \frac{\langle S \rangle \cdot R}{\langle S_{ho} \rangle} = \frac{8,35 \cdot 10^3 \cdot 3,24}{2,7 \cdot 10^3} = 100,2 \text{ cm}^3. \quad (19)$$

With the previous value $b = 2\text{cm}$, let us determine the new value of the cross-section $W = bh^2 / 6 = 2 \cdot h^2 / 6 = 0,333h^2$ height h , taking into account expression (19):

$$h_1 = \sqrt{\frac{100,2}{0,333}} = 17,35 \text{ cm}. \quad (20)$$

A newly designed cross-section is assigned in accordance with expression (20) ($b \times h = (2 \times 18\text{cm})$) instead of the previous cross-section ($b \times h = (2 \times 5\text{cm})$) in order to ensure the required load-bearing capacity of the structure considered in Figure 1.

The performed probabilistic calculation showed that the initial beam cross-section does not provide the required reliability and load-bearing capacity under the specified operating conditions. The results indicate that, during long-term operation and at the adopted level of probability of failure-free operation, the existing geometric parameters of the section are insufficient to resist the applied loads. As a result of the probabilistic calculation, the required parameters of the strengthened cross-section were determined, ensuring the necessary strength reserve and reliability of the structure. It was established that increasing the beam height can significantly increase the load-bearing capacity of the element without changing its width. The adopted new section satisfies the strength conditions and may be recommended for further design and operation of the structure under consideration.

Let us apply the theoretical provisions presented above for assessing load-bearing capacity based on the outlier theory to the case of statistical technical-inspection data for a conditional reinforced concrete structure. The use of a probabilistic approach makes it possible to account for the random nature of changes in external actions, material strength characteristics, and geometric parameters of the structure that affect its operational reliability. Unlike deterministic calculation, statistical analysis makes it possible to assess the probability of reaching a limiting state during operation [29-30].

For the calculation, statistical characteristics of the main factor parameters obtained from inspection results and operational-data analysis were used. Data processing was performed by determining mean values, standard deviations, and coefficients of variation, which makes it possible to assess the influence of each parameter on the load-bearing capacity of the structure. Statistical data for the object under consideration are given in Table 1.

Table 1. Statistical characteristics of factor parameters and load-bearing capacity of the reinforced concrete structure

No.	X_1 , (MN/m)	X_2 , (MPa)	X_3 , (MPa)	X_4 , (m)	Y , (MPa)
1	0,04	240	15	0,8	10
2	0,05	210	17	0,6	12
3	0,045	220	20	0,7	11
4	0,055	230	22	0,9	9
5	0,06	215	16	1,0	13
Sum	0,25	1115	90	4,0	55
Mean	0,05	223	18	0,8	11
σ	0,006	10,7704	2,6077	0,2289	1,2
ν	0,12	0,0483	0,1449	0,28613	0,1091

The use of the probabilistic outlier method makes it possible to assess the degree of dispersion of random variables relative to their mean values and to determine the influence of variable loading factors on the reliability and load-bearing capacity of the structure during a specified operating period. The statistical characteristics were determined on the basis of processed experimental and calculated data, which enables further probabilistic analysis of structural serviceability.

Table 1 presents the factor parameters: X_1 is the variability of external load; X_2 is the design resistance of reinforcement; X_3 is the design resistance of concrete; X_4 is the effective depth of the cross-section; and the resulting parameter Y is the load-bearing capacity of the reinforced concrete structure.

Based on known methods of variance and correlation analysis, probabilistic characteristics of parameter X_1 , reflecting the variability of the external load acting on the reinforced concrete structure, were obtained below [6, pp. 134-140]. The standard deviation is indicated accordingly $\sigma(X_1) = 0,006 \text{ MN} / \text{m}$; $\sigma(Y) = 1,2 \text{ MPa}$.

Operating time of the structure:

$$T = 40 \text{ years} = 40 \cdot 24 \cdot 365 = 1,26144 \cdot 10^9 \text{ sec.} \quad (21)$$

Normative (specified) probability of failure-free operation:

$$[p] = 0,95. \quad (22)$$

According to Table 1, the mathematical expectations at equal probability $p_i = 0,2 = \text{const}$ are:

$$M(X_1) = \sum p_i X_{1,i} = 0,2(0,04 + 0,05 + 0,045 + 0,055 + 0,06), \quad M(X_1) = 0,05 \text{ MN} / \text{m}. \quad (23)$$

$$M(Y) = \sum p_i Y_{1,i} = 0,2(10 + 12 + 11 + 9 + 13), \quad M(X_1) = 0,05 \text{ MN} / \text{m}. \quad (24)$$

Variations of the corresponding parameters:

$$\nu_{X_1} = \frac{\sigma(X_1)}{M(X_1)} = \frac{0,006}{0,05} = 0,12; \quad \nu_Y = \frac{\sigma(Y)}{M(Y)} = \frac{1,2}{11} = 0,1091. \quad (25)$$

The problem was solved with allowance for expressions (21)-(25). The ratios of the standard deviation values $\sigma_{ho} = 0,8 \text{ MPa}$ were determined for the adopted conditional value.

$$\varphi = \frac{\sigma(Y)}{\sigma_{ho}} = \frac{1,2}{0,8}; \quad \varphi = 1,5 \quad (26)$$

Using expression (11), the value Q was determined:

$$Q = 1,5(1 + 1,5^2)^{-1/2}; \quad Q = 4,125 \quad (27)$$

Using formula (10), the safety characteristic was determined:

$$\gamma = -2 \ln \left(\frac{0,0513 \cdot 10^{-7}}{26,0172} \right)^{1/2}; \quad \gamma = 4,7273 \quad (28)$$

Using expression (12), the values of the strength reserve factor were obtained with allowance for expressions (25), (26), and (28) [31-33]:

$$K = \frac{[1,5 + 4,7273 \cdot 0,1091 \cdot (1 + 1,5^2)]^{1/2}}{1,5} = 1,62; \quad K = 1,62. \quad (29)$$

Using formula (13), the strength condition ($[p] = 0,95$) was checked with allowance for expressions (23) and (24):

$$\frac{M(y_0)}{R} \geq M(y), \quad (30)$$

where $M(y_0) = 20 \text{ MPa}$ the parameter denotes the mathematical expectation of the dangerous limiting stress.

Using expression (30), it was determined that $\frac{200}{1,62} \geq 11$ or $12.346 > 11$, therefore, the strength condition is satisfied under the probabilistic calculation method.

Using expressions (1)-(3), the lower estimate of the probability of failure-free operation was determined for the initial data of Example 1 considered in this article:

$$b = 1,26144 \cdot 10^9 (5 \cdot 10^{-2}) \left(1 + \frac{0,8^2}{1,2^2} \right)^{-1/2} \exp \left[(20 - 11)^2 : 2 : (1,2^2 + 0,8^2) \right] = 0,17862; \quad (31)$$

$$b = 0,17862.$$

Using formula (1), and taking into account the value obtained in (31), the following was determined for 40 years of actual operation at the time of inspection:

$$p_{ho}(40 \text{ years}) = 1 - 0,17862 = 0,82138; \quad p_{h.o} = 0,82138. \quad (32)$$

According to value (32), the probability of failure-free operation of the structure under consideration (Figure 2) over 40 years of operation is 82%.

The failure intensity over 40 years of long-term operation, with allowance for (32), was determined as follows [34]:

$$\lambda = \frac{\ln(p_{ho}(40))}{-T} = \frac{\ln(0,82138)}{-40} = 0,00492 \text{ year}^{-1}. \quad (33)$$

The time percentage gamma-life was determined considering expression (33):

$$t_{\gamma} = \frac{\ln(p_{no}(40))}{-\lambda} = \frac{\ln(0,82138)}{-0,00492} = 40 \text{ years.} \quad (34)$$

The calculations showed that the proposed probabilistic approach, based on the theory of probabilistic outliers, makes it possible to quantitatively assess the reliability of long-term operating building structures while accounting for the statistical variability of loads and load-bearing capacity. Based on the statistical characteristics of the factor parameters of the reinforced concrete structure, the safety characteristic $\gamma = 4.7273$ and the strength reserve factor $K=1.62$ were determined. The strength check according to expression (30) confirmed satisfaction of the reliability criterion at the specified normative probability of failure-free operation $[p] = 0.95$.

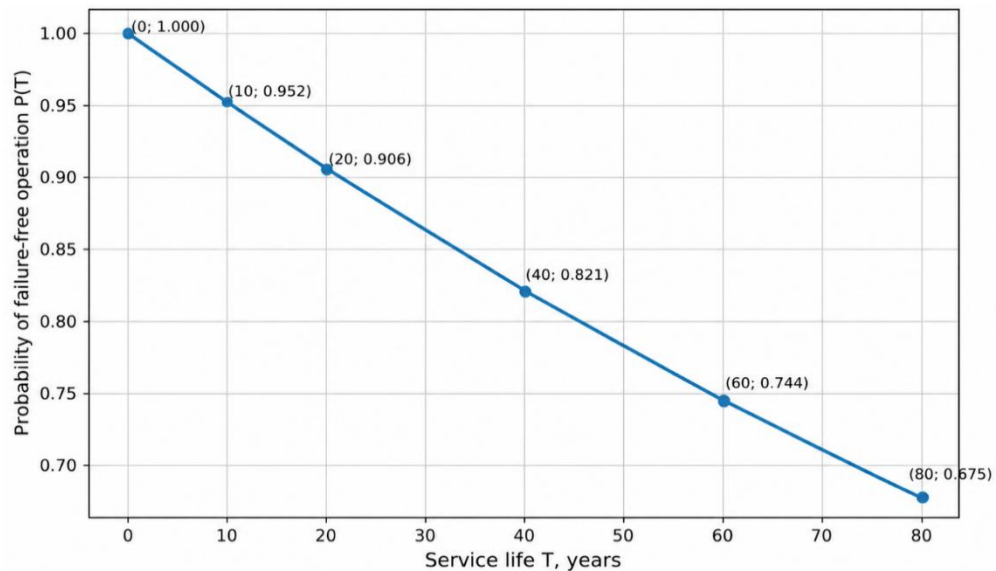


Fig.2. Dependence of the probability of failure-free operation of the structure on service life

In addition, the actual probability of failure-free operation of the structure over 40 years of service was determined as $P_{no}=0.82138$, which corresponds to approximately an 82% probability of failure-free operation. The results confirm that the application of the theory of probabilistic outliers makes it possible to perform a probabilistic assessment of operational reliability and to predict the residual service life of long-term operating building structures.

Based on the results of the probabilistic analysis, a dependence of the probability of failure-free operation of the structure on the service life was obtained (Figure 2). The calculation was performed using the obtained failure intensity $\lambda = 0.00492$ year⁻¹ according to expression (33).

The approximating function for the dependence of the probability of failure-free operation shown in Figure 2 has an exponential form [35-36]:

$$P(T) = e^{-\lambda T},$$

where $P(T)$ is the probability of failure-free operation of the structure; λ is the failure intensity; T is the service life of the structure, years.

Taking into account the failure intensity $\lambda = 0.00492$ year⁻¹ obtained in (33), the approximating function takes the following form (Figure 2):

$$P(T) = e^{-0,00492T}. \quad (35)$$

The obtained dependence describes an exponential decrease in the probability of failure-free operation of the structure over time. At $T=40$ years, the value of the function $P(40) = e^{-0,00492 \cdot 40} = 0,82$ corresponds to the result of the probabilistic calculation performed above. The obtained function can be used to predict residual service life and assess changes in the reliability of building and mechanical-engineering structures during long-term operation.

Analysis of the graph shows that the probability of failure-free operation of the structure decreases according to an exponential law as service life increases. At the initial stage of operation, the decrease in probability is relatively

slow; however, during long-term service, the influence of random loads, damage accumulation, and material degradation gradually reduces the reliability level [37-39].

The characteristic function values are as follows: at a service life of 0 years, the probability of failure-free operation is 1.0; at 10 years, 0.952; at 20 years, 0.906; at 40 years, 0.821; and at 60 years, 0.744.

For the design service life of 40 years, the probability of failure-free operation is about 0.82, which agrees with the results obtained in this study. The constructed dependence confirms the possibility of applying the theory of probabilistic outliers to predict residual service life and assess the operational reliability of building and mechanical-engineering structures [40].

Conclusion

The paper presents a probabilistic method for assessing the reliability of building structures during long-term operation. The method is based on the theory of outliers of random processes and accounts for the actual scatter of loads, strength characteristics of materials, and parameters of the stress-strain state. The proposed algorithm analyzes the frequency with which a random load exceeds a critical level, making it possible to move from standard deterministic strength calculations to a direct quantitative assessment of object reliability.

Analytical relationships were obtained that link the probability of failure-free operation, safety index, reserve factor, and statistical parameters of the load-resistance system. Unlike classical calculations according to building standards, where fixed loads and normative reliability factors are used, this approach accounts for the real statistics of an object accumulated over years of service, including material wear and changes in operating conditions. This is critically important when inspecting old buildings whose actual condition is already far from the original design state.

Using the developed methodology, a steel beam and a reinforced concrete structure were checked. The calculation based on outlier theory helped identify a deficit in load-bearing capacity, determine strengthening parameters, and calculate reliability at the specified normative probability of failure-free operation. For the steel beam, it was established that the original section did not provide the required reliability; therefore, new safe profile dimensions were selected. For the reinforced concrete structure, based on actual inspection data, the probability of failure-free operation over 40 years of service was determined as $P_{no} = 0.82138$, and the failure intensity and gamma-life were calculated.

The analysis showed that the decrease in the probability of failure-free operation of the structure over time follows an exponential law. This confirms the classical nature of the decline in the operational reliability of materials and the accumulation of damage during long-term service. The constructed approximating function clearly illustrates the decline in reliability as service life increases. The obtained dependence makes it possible to forecast the condition of a structure for years ahead, assess its residual service life, and calculate the limiting periods of safe operation while accounting for operational loads.

The results confirm that probabilistic analysis describes the influence of scatter in loads and material properties on structural reliability much more accurately than rigid deterministic calculations. From an engineering standpoint, this makes it possible to make more balanced and economically justified decisions when assessing technical condition, designing strengthening measures, carrying out reconstruction, and predicting the residual service life of objects.

The practical value of the work is that the methodology can be directly applied to the inspection of operating steel structures, reinforced concrete structures, bridges, and industrial buildings subjected to random loads. The transition to a probabilistic approach makes it possible to replace the formal fulfillment of regulatory requirements with an accurate quantitative assessment of the safety of a specific object using field monitoring data.

The developed outlier-theory apparatus can be regarded as a promising tool for the further development of methods for calculating building structures. In the future, these algorithms may form the basis for creating digital systems for structural monitoring, durability prediction, and intelligent models for managing the safety of buildings and facilities.

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