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Dynamics of the Vibration Process of a Pumping unit Taking into Account the Elastic and Dissipative Properties of the Elements of Support Devices

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Abstract. The dynamics of the vibration process of a pumping unit can be considered as independent oscillations of two separate sprung masses from the centrifugal force: the rotor on the bearings and the pump casing on the foundation supports. In the absence of special damping devices in the oscillating system, the reaction force from the rotor vibration affects the casing and the bearing supports approximately equally. However, when dampers are used in the oscillating system, they can significantly reduce transmitted force to unit supports. Such a method made it possible, in addition to the acoustic parameters of the unit, to determine the dynamic loads on the supports and to conduct a comparative assessment of the strength parameters of rolling bearings. Vibration protection of rotating machines, i. e. with a rotor, which includes an impeller with a shaft and support parts, requires a large amount of research, largely experimental in nature. At present, some of this work can be significantly reduced by using mathematical apparatus to perform calculations and model dynamic processes. The aim of the paper is to show method and model to calculate dynamics of vibration process in a pumping unit, considering elastic and dissipative properties of elements in support devices.

Keywords: damping, vibration, shock absorbers, frequency, rotor.

Introduction.

An effective method of vibration isolation is to provide the pump with elastic and dissipative connections, for example, on the pump mounting supports to the frame. The problem is that the methods of calculation and assessment of the effect of damping devices on reducing vibration emitted by pump units existing in modern literature are not accurate enough. They are performed without taking into account the inertia of oscillating masses and internal friction in the support materials. More accurate calculations can be made using nonlinear differential equations describing the processes of mass oscillation, i.e., methods of mathematical modelling of the process dynamics. The paper considers one of the main types of vibration of pump units arising due to rotor imbalance when the shaft axes are displaced. An assessment is made of the effect of operating pumps with elastic connections made of elastomeric materials on the dynamic loading of the supports. It is established that vibrations will decrease when using damping devices made of modern high-tech elastic materials in the form of elastomers. Polyurethane elastomers are produced in Austria by Getzner Werkstoffe GmbH under the Sylomer and Sylodyn brands and can be recommended for vibration isolation of pump units. The aim of the paper is to show method and model to calculate dynamics of vibration process in a pumping unit, considering elastic and dissipative properties of elements in support devices.

1. Materials and methods

Isolation of pump vibrations with shock absorbers achieves the goal when the ratio of the frequencies of forced and natural vibrations of the mechanical system,

$$\omega/\omega_{\mathrm{a}} > \sqrt{2}.$$

where ω - forced frequency, ω_{a} - the natural frequency.

In practice, the ratio of these frequencies is taken as 2.5–5.

The transfer coefficient at low damping can be calculated by the following formula

$$\mu_t = \left| 1/1 - (\omega/\omega_{\mathrm{a}})^2 \right| \tag{1}$$

Effective vibration isolation is attained with $\mu_t = 0.125 - 0.066$. The vibration isolation (dB) of the shock-absorbing mount is defined by the reciprocal of the transmission coefficient, represented in logarithmic units:

$$\Delta L = 20\, lg|F/F_a| = 20\, lg|1 - (\omega/\omega_a)^2| \tag{2}$$

To ensure acceptable levels of noise and vibration, the effectiveness of the unit's acoustic vibration insulation ΔL should not be less than 26, for centrifugal pumps given at [1].

In the paper [2] it is demonstrated that the research of vibrations occurring on supports of the rotor shaft in the housing and in the pump mounts on the foundation can be considered as independent vibrations of two separate sprung masses. In this regard, in this work a study of two vibration variants is carried out:

1) the rotor on bearings from centrifugal force, which results from the pump and electric motor shaft axes moving parallel to the centre axis;

2) For the same reason, the foundation's pump housing is supported by centrifugal force, but without taking into account the elastic-damping properties of the bearings.

It should be noted that the assessment of vibration isolation efficiency of pumps is currently carried out using the second calculation option. However, to what extent this is justified in detailing vibration studies can be considered using the example of vibration calculations of the 2D630-125 pump. These units are used at industrial water supply pumping stations, as well as in the petrochemical industry. A parallel offset of the pump and electric motor shaft axes relative to the central axis of 0.10 mm was taken. The centrifugal force on the supports of rotor, with mass of rotor $m_r = 120\ kg$ and shaft speeds $\omega_s = 308.7 s^{-1}$ makes up

$$F_a = 120 \cdot 308,7^2 \cdot 0,1 \cdot 10^{-3} = 1145\ N \tag{3}$$

To simplify calculations, the elastic and dissipative properties of the elements of the oscillatory system are taken into account by constant coefficients. For example, the rigidity of rolling bearings, represented through the static load capacity W_0**,** can be determined by the formula [3]

$$k_n = 5350 \sqrt[3]{d_r W_0 z^2 cos^2 \psi}\ , \tag{4}$$

where d_r– diameter of rolling elements;

z – number of rolling elements;

ψ – nominal contact angle.

However, it gives an overestimated value of the bearing stiffness coefficient, since it is calculated for the static loading of all rolling elements and does not take into account the uneven distribution of the load between them. The following formula is suggested in [2]

$$k_{rb} = F_0/a \tag{5}$$

As per the theory of load distribution for rolling elements in single bearing, the ball that is opposite the load's line of action experiences the most force, which as following

$$F_0 = 5\ F_a/2z, \tag{6}$$

The convergence of the axes of two spheres in contact along concave surfaces according to Hertz's theory [3]

$$a = 1,55 \sqrt[3]{(F_0/E)^2 (R_1 - R_2)/2R_1 R} \tag{7}$$

Natural frequency of the system

$$\omega_a = \sqrt{2k_{rb.}/m_r} \tag{8}$$

The ratio of the frequencies of the disturbing force oscillations and the system's natural oscillations of $308.7/1053 = 0.292$. $[1 - (\omega/\omega_a)^2\ ] = 1 - 0.292^2 = 0.914$ and transfer coefficient $1/0.914 = 1.09$ not within the suggested range

$\mu_t = 0.125 - 0.066$, which characterises the low vibration isolation of bearings. To ensure isolation of rolling bearings from vibration, the frequency of the system's natural oscillations must be decreased and support compliance must be increased. This problem can be solved in one of several ways:

- structurally by installing a polyurethane elastomer bushing on the bearing seating surfaces in the pump housing;
- technologically - by spraying a polyurethane layer on the same surface;
- applying a film of composite material and dispersed metal additives [4].

The last two methods are limited by the thickness of the sprayed or applied layer δ, with an increase in which its homogeneity and strength decrease. A decrease in thickness helps to increase the rigidity k of the mounting surface and, accordingly, the frequency of the natural oscillations of the system. Figure 1 shows a diagram of installing a polyurethane bushing in a hole in the pump body for mounting a bearing in it. For instance, motorists mostly use polyurethane shock absorber bushings to repair vehicles. Main properties of polyurethane bushings are presented below:

- wear resistance;

- low friction;
- elasticity;
- operation at high pressure;
- long service life;
- resistance to oils, fuel and petroleum.

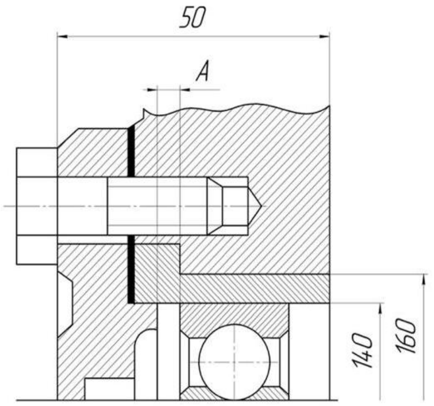

Fig. 1 – Installation of a polyurethane bushing into the hole of the 2D630-125 pump body

Rigidity coefficient of polyurethane bushing

$$k_{el.} = \mathrm{EA}/\,\delta, \tag{9}$$

where $\mathrm{A} = \pi\mathrm{B}\,(D\; +\; \delta\,)$– surface area of polyurethane bushing;
δ – polyurethane bushing thickness;
B – bearing width;
D – diameter of the bearing bushing hole.

The wall thickness of a polyurethane bushing is determined by the Bach formula

$$\delta = \frac{\mathrm{D}}{2}\left(\sqrt{\frac{[\sigma_\rho]+0.4\mathrm{P_Z}}{[\sigma_\rho]-1.3\mathrm{P_Z}}} - 1\right), \tag{10}$$

where σ_ρ – the permissible tensile stress of elastomer grade SR55, in accordance with the data of the "Getzner Werkstof" company [5] is equal to 0.061 MPa;
p_z – the pressure on the bushing's walls is calculated as F_a/A.

When connecting in series the elements of a viscoelastic system consisting of a bearing and a polyurethane bushing, the reduced stiffness

$$k_{re} = k_n k_{el}\;/\;(k_n + k_{el}). \tag{11}$$

For damped oscillations of systems with damping, the natural frequency is determined by the frequency of free oscillations ω_0

$$\omega_a = \sqrt{\;}\,\omega_a^2 - n^2, \tag{12}$$

where $n = \omega_0\psi/4\pi$– the given damping coefficient through the energy loss coefficient ψ connects the oscillation frequencies of the two systems in such a way

$$\omega_a = \omega_0\sqrt{1} - (\psi/4\pi)^2 \sim \omega_0, \tag{13}$$

where ω_0 - the frequency of free oscillations.

Due to the insignificance of the damping effects, the natural frequency of a system consisting of a more flexible support is determined by the well-known formula

$$\omega_a = \sqrt{2} k_{re}/m = \sqrt{\,2 \cdot 0.364 \cdot 10^6/120} = 77\ s^{-1} \tag{14}$$

The ratio of the frequencies of the oscillations of the disturbing force and the natural oscillations of the system $308.7/77 = 4$. Expression $|1 - (\omega/\omega\_a\,)^2| = |1 - 4^2| = 15.$ Transfer coefficient $1/15 = 0{,}066$.

Based on computations

$$\Delta L = 20\ lg 1/0.066 = 26.02\ Hz, \tag{15}$$

that corresponds to the permissible level of vibration. Thus, the use of a bearing support with a flexible base in the form of a polyurethane bushing provides sufficient acoustic vibration insulation of the unit.

To assess the dynamic properties of the system, forced vibrations of the viscoelastic model shown in Figure 2 are considered. Taking into account the elastic forces *kx* and linear viscous resistance *bx*, the differential equation of motion of the mass m has the form

$$m\ddot{x} + b\dot{x} + kx = F\,sin\,\omega\,t, \tag{16}$$

or

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F\sin\omega t$$

where m is the mass, b is the damping coefficient, k is the stiffness, x is the displacement, F is the forcing amplitude, and ω is the forced frequency.

Through the reduced damping and mass

$$b = 2\,nm = 2m(\omega_0\psi/4\pi) = m\omega_0\psi\,/2\pi \tag{17}$$

The final expression is obtained

$$b = k_{el}\psi/2\pi\omega_0, \tag{18}$$

where ψ – coefficient of mechanical losses of the elastomer, for the previously selected elastomer brand SR55 – $\psi = 0.17$.

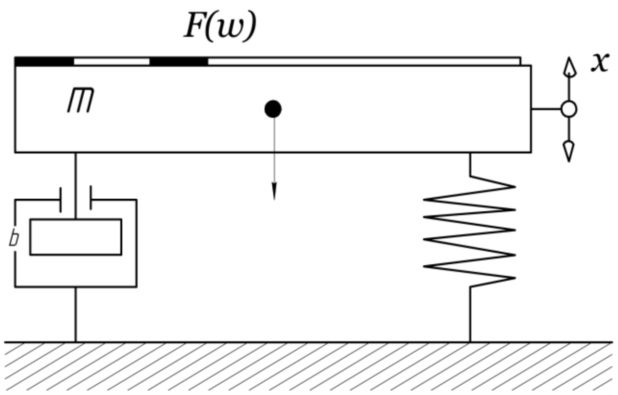

Fig. 2 - Model of a rotor on bearings in the form of a viscoelastic body

Bushing settlement from the rotor weight per support

$$x_{cm} = \frac{mg}{2k_{el}} = \frac{1200}{2} \cdot 0.364 \cdot 106 = 0.001648\,m \tag{19}$$

The differential equation of motion of the mass m, taking into account the settlement of the walls of the polyurethane bushing, takes the form

$$m\ddot{x} + b\dot{x} + k(x + \mathrm{x}_0) = F\,sin\,\omega\,t \quad (20)$$

The elastic and damping properties of the components in this model must be known in order to replicate the vibrations of the pump body on the foundation [9]. A concrete class of at least 10 (compressive strength in MPa) is required in order to construct a foundation for a 2D630-125 pumping unit. Class B 25 concrete is approved, and the pumping unit's foundation is rigid.

$$k_f = EA_f/h, \quad (22)$$

where $E = 30 \cdot 10^3$ MPa – modulus of elasticity of concrete under compression;

A_f – the foundation base's area is taken to be larger than the area of the horizontal projection of the pump;

h – the underground part's depth of the foundation, for heated rooms the minimum - 0.5 m

$$k_f = 30 \cdot 10^9 \cdot 0.33/0.5 = 20 \cdot 10^9 N/m. \quad (23)$$

System's natural frequency

$$\omega_a = \sqrt{20000} \cdot 10^6 /1350 = 3874\,s^{-1} \quad (24)$$

significantly exceeds the frequency of disturbing vibrations, that characterises system's low vibration isolation. $[1 - (\omega/\omega_a)^2\,] = [1 - (308.7/3874)^2] = 1$. Transfer coefficient μ_t=*1*.

Stress occurring in elastomer flooring with unit weight 13500 N

$$\sigma = 13500/0{,}2232 = 60483 N/m^2 \quad (25)$$

Elastomer flooring settlement from pump weight

$$x_s = H \cdot \sigma/E, \quad (26)$$

where H – the thickness of the flooring is taken as 0.4 m;

For grade SR55 elastomer, the dynamic modulus of elasticity - 0.753 MPa

$$x_s = 0.04 \cdot 60483/0.753 \cdot 10^6 = 0.003212\,m \quad (27)$$

Rigidity of flooring under the pump sole

$$k_{el} = 0.753 \cdot 10^6 \cdot 0.2232/0.04 = 4.2 \cdot 10^6 \mathrm{N/m} \quad (28)$$

Natural frequency of pump oscillation on elastomer flooring

$$\omega_a = \sqrt{4.2} \cdot 10^6/1350 = 55.67 s^{-1} \quad (29)$$

The ratio of frequencies of disturbing force's oscillations and the natural oscillations

$$308.7/55.67 = 5.54. \quad (30)$$

$$|1 - (\omega/\omega_a)^2| = |1 - 5.54^2| = |29.69|. \quad (31)$$

Transfer coefficient $1/29.69 = 0.033$. Since the rigidity of the foundation is much higher than the rigidity of the elastomer, the rigidity of the decking is taken as the design rigidity. Damping coefficient

$$b_{el} = k_{el}\psi/2\,\pi\omega = 4.2 \cdot 10^6 \cdot 0.17/6.28 \cdot 55.67 = 2042\ \mathrm{Ns/m} \quad (32)$$

2. Results and discussion

The results of calculations using known methods for calculating differential equations [6] with different values of the stiffness and damping coefficients are presented in Table 1. The calculation is based on the representation of a non-homogeneous equation through a characteristic equation, then using the method of varying constant coefficients, a general solution is found in which the dependence of the mass displacement on time is determined in analytical or graphical form.

The elastic resistance force on the supports is calculated based on the amplitude of the mass displacement, taking into account damping $R = kx + bx^{/}$. The damping rate is determined by the formula

$$x^{/} = fx, \tag{21}$$

where f – frequency of oscillation of pump rotor on supports, Hz

Table 1. Results of calculations with different values of stiffness and damping coefficients

Number of row	m, kg	k, N/m	b, Ns/m	x, m	R, N	μ_t	k_n
1	60	$65 \cdot 10^6$	-	0000096	624	0.97	0.97
2	60	$364 \cdot 10^3$	125	-0.00021	86.4	0.066	6.62
3	1350	$20 \cdot 10^9$	-	$5.85 \cdot 10^{-8}$	1170	1.0	1.0
4	1350	$4200 \cdot 10^3$	2042	-0.00002	84	0.033	13.6

The ratio of the exciting force F_a to the resistance force R is used to determine the force transfer coefficient k_n, which in oscillatory systems without damping is equal to the transfer coefficient μ_t [7], [8]. In mechanical units where devices with built-in dampers are used, these coefficients may differ significantly. In the absence of damping, the reaction in the bearings falling on one side of the rotor, $R = 624$ N, is approximately equal to half the load from the exciting force $F_a = 1145/2 = 572.5N$. The reaction in the pump casing supports $R = 1170\ N$ with a small difference - the force exciting the rotor oscillations $F_a = 1145$ N. In pumps with dampers, the above regularities are not observed. In the presence of dampers, the reaction force on the bearings is less than the exciting force falling on it by $572.5/86.4 = 6.62$ times. The reaction on the pump body supports is $1145/84 = 13.6$ times.

Conclusions

The analysis of the results of the calculation shows that., vibration of the mass without taking into account damping is of a harmonic nature (row 1 and 3 in Table 1), as well as the oscillations of the disturbing force, they have positive and negative values of the amplitudes of the cycles. The reaction forces are approximately equal to the load exciting the oscillations, i. e. $F_a \sim R$, which is possible when ratio of frequencies of disturbing and natural oscillations is insignificant because of high frequency of the latter. Considering shock absorbers' damping and static settlement, the vibration is a pulsating (row 2 and 4 in Table 1) curve, the amplitude is determined as the difference in the ordinates of the peaks: with a negative result, the direction of mass movement is against the gravity vector of the pump, i. e. aimed at weakening the initial settlement of the shock absorber.

When using a damping base under the pump body, the stress in the threaded connection during vibrations is reduced as a result of reducing the calculated external load. The article presents a method for calculating the dynamics of the vibration process of a pump unit taking into account the elastic and dissipative properties of the elements of the support devices. In the absence of special damping devices in the oscillatory system, the reaction force in the supports is equal to the load exciting the vibrations, which is possible with high rigidity and frequency of the latter. The reaction forces from the rotor vibration affect the housing and the bearing supports approximately equally. However, when using dampers in the oscillatory system, they can significantly reduce the force transmitted to the unit supports. The reaction forces on the bearings must be determined by detailing the pump model, since, as established above, they can differ significantly from the disturbing force.

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