DOI 10.52209/2706-977X_2025_2_31 IRSTI 55.03.14 UDC 621.92

Reducing Vibration of Elastically Supported Rolling Bearings on a Centrifugal Pump

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Abstract. In this work, the vibration of the pump rotor supports from centrifugal force is studied. For this purpose, it is proposed to install a flooring made of vibration-insulating material under the pump casing. When fitting the outer rings of the rolling bearing into the pump casing through an elastic bushing, it is possible to increase the compliance of the supports while maintaining sufficient strength and wear resistance of the surfaces. It is proposed to use such a complex support of the pump rotor, consisting of a rolling bearing and a polyurethane bushing, which allows, in case of rotor imbalance caused by the radial displacement of the axes of the pump and the engine, to reduce vibrations at the main oscillation frequency by several times. On the casing supports, when using vibration-insulating material as a flooring, it is significantly reduced by 5.87 times. And the use of elastic elements made of elastomer as a flooring for the pump casing and the installation of a damper made of a polyurethane bushing under the outer ring of the rolling bearing reduce vibration loads perceived by the foundation and rolling bearings by tens of times.

Keywords: elastically supported bearing, vibration, pump, elastomer, rotor.

Introduction

The article uses the experience of foreign sources, in particular, the author Jerry Hallam published the results [1] of a large-scale study of the reliability of 480 pumps over a five-year period at the Amoco Texas City oil refinery. The author's National Instrument Lab VIEW system [2] allows you to evaluate the tested machine based on the signals generated by the machine, and polyurethane-based damping materials developed by the Austrian company Getzner Werkstoffe GmbH [3] allow you to absorb vibrations in a wide range of frequencies and loads. Vibration and process data were collected on individual single-stage centrifugal pumps from a number of industrial enterprises in the Perth area [4]. Considering the main causes of vibration of the pumping unit, we determined the frequency spectrum by which it is necessary to select the amplitude-frequency characteristics of the damping devices and optimize their parameters. Unbalance of the rotor (imbalance) during its rotation causes variable loads on the rotor supports and shaft bending. This is the most common of the frequently encountered causes of vibration. The maximum amplitude of vibration during imbalance has a radial direction. The vibration frequency (in Hz) typical of imbalance is: f = n/60, where n is the rotor speed per minute. Rolling bearing defects are characterized by a high-frequency component, for most possible defects with a frequency equal to f and multiples of it by the number of rolling elements. The operation of technological and chemical plants at reduced capacity led to the fact that highly efficient or large-sized pumps were operated in a wide range of performance, including long-term operation at flows significantly below capacity [5]. Shock-free throughput without vibration is considered the most important indicator of the best operating conditions [6]. According to authoritative sources who examined several dozen pumping station sites of main oil pipelines, about 38-45% of all failures occur due to increased vibrations [7]. They cause bearing failure, dynamic loading of the pump-to-foundation connections, rotor displacement and beating. In this regard, the operating conditions of the end and slot seals deteriorate, accompanied by an increase in fluid leaks and overlapping of gaps in the casing and impeller connections. The main causes of vibration problems in a single-stage and multi-stage centrifugal pump are synchronous vibrations, mechanical imbalance, etc. It is known that a pump unit on a frame with elastic damper supports experiences 50 times less vibration loads. However, these shock absorbers primarily solve the problem of reducing vibration transmitted to the foundation, and also require significant time and money to perform repairs and replace them. In the work [8] it is shown that the study of vibrations occurring on the pump rotor shaft supports and in the pump casing fastenings on the frame can be considered as vibrations of two separate masses with elastic supports. In particular, it was found that vibrations excited by the imbalance of the pump rotor are compensated to a greater extent by using damping on the supports themselves. In particular, these conclusions are confirmed by research by Getzner Werkstoffe (Austria) [3].

The purpose of this work is to study the vibration of the rotor supports from the centrifugal force caused by the displacement of the rotor relative to the central axis of the 2D630-125 Pump. Pumps 2D630-125 and units based on them are used at pumping stations of urban, industrial and rural water supply, as well as in the petrochemical industry.

1. Methods and experiment

Frequency of vibrations during shaft rotation

$$f = k \cdot \varphi / 2\pi, \tag{1}$$

where $k = 1, 2, 3$ – harmonic number.

At $k = 1$ for the fundamental frequency $f = 308{,}7/6{,}28 = 49{,}15\ Hz$, the maximum permissible parallel displacement of the axes is 0,10 mm. The centrifugal force acting on the rotor supports, with a rotor mass of $m_r = 120\ kg$ is

$$F_a = 120 \cdot 308{,}7^2 \cdot 0{,}1 \cdot 10^{-3} = 1145\ N$$

The main load in the existing formulae for calculating the rigidity coefficient for assembled rolling bearings is taken to be their load-carrying capacity, i. e. the actual load and the pattern of its distribution on the rolling elements are not taken into account. However, the greatest force falls on the ball [2], located opposite the line of action of the load. Its value is approximately equal to

$$F_0 = 5F/z, \tag{2}$$

where z – number of rolling elements for a bearing;

F – the radial force supported by one bearing, equal to half the sum of the rotor gravity and centrifugal forces

For a centrifugal pump 2D630-125 with a two-way supply of liquid to the impeller, the radial force on the bearing

$$F = (120 \cdot 9{,}8 + 1145)/2 = 1172{,}5\ N$$

then according to the formula for calculating the greatest pressure force with the number of rolling elements for bearings №313 – $z = 5$

$$F_0 = 5 \cdot \frac{1172{,}5}{8} = 732{,}5\ N$$

The load distribution depends to a large extent on the size of the bearing clearance and on the accuracy of the geometric shape. The convergence of the axes of two spheres contacting along concave surfaces according to Hertz's theory [2]

$$\delta = 1{,}55 \sqrt[3]{(F_0/E)^2 (R_1 - R_2)/2R_1R_2}, \tag{3}$$

where R_1 – radius of curvature of the surface of the inner ring along the racetrack

$$R_1 = ((D - d)/2 - d_b)/2 + 0{,}5d = ((140 - 65)/2 - 23{,}8)/2 + 0{,}5 \cdot\ 65 = 46{,}2\ \text{mm}$$

$R_2 = 12\ mm$ – radius of curvature of the ball. Then, according to the formula, the convergence of the axes of the two spheres during deformation of the rolling elements

$$\delta = 1{,}55 \sqrt[3]{(732{,}5/2 \cdot 10^5)^2 (46{,}2 - 12)/2 \cdot 46{,}2 \cdot 12} = 0{,}011\ mm$$

Rolling bearing rigidity

$$c_{r.b.} = F_0/\delta = 732{,}5/0{,}011 \cdot 10^{-3} = 66{,}59 \cdot 10^6\ N/m$$

Natural frequency of an oscillatory system ω_a

$$\acute{\omega}_a = \sqrt{2c_{r.b.}/m_r},$$

where m_r – rotor mass;

$c_{r.b.}$ – rigidity of rolling bearings on both sides of the rotor.

$$\acute{\omega}_a = \sqrt{2 \cdot 66{,}59 \cdot 10^6/120} = 1053\ s^{-1}$$

The transfer coefficient [3] at low damping can be approximately determined by the formula

$$\mu_t = \left|1/1 - (\omega/\omega_a)^2\right|, \tag{4}$$

The amplitude of the force transmitted in the direction of the vertical axis is determined by the formula

$$F = F_a \cdot |1 - (\omega/\omega_a)^2|, \tag{5}$$

where F_a – amplitude of the disturbing force;
F – amplitude of force transmitted to the support

The ratio of the frequencies of the oscillations of the disturbing force and the natural oscillations of the system $308{,}7/1053 = 0{,}292$. Expression $\left|1-(\omega/\omega_a)^2\right| = 1-0{,}292^2 = 0{,}914$.

The transfer coefficient at low damping can be approximately determined by the formula:

$$\mu_t = \left|1/1-(\omega/\omega_a)^2\right|, \tag{6}$$

Good vibration isolation is achieved with $\mu_t = 1/8 - 1/15 = 0{,}125 - 0{,}066$.

The transmission coefficient $1/0{,}914 = 1{,}094$ is not included in the recommended range, which characterizes low vibration isolation of bearings. The force transmitted to the support without taking into account the damping of the system F $F = 1145 \cdot 1{,}094 = 1252$ N slightly exceeds the value of $1252/1145 = 1{,}09$ times that exciting from centrifugal forces. To ensure vibration isolation of rolling bearings, it is necessary to increase the compliance of the support and reduce the frequency of natural oscillations of the system. It is proposed to install a polyurethane bushing in the hole of the 2D630-125 pump housing, instead of the bearing cup. In this case, all technical requirements and dimensions adopted for assembling the bearing in the housing are preserved.

Polyurethane bushings are currently used in cars, for example, polyurethane shock absorber bushings are widely used by motorists for vehicles. The main advantages of polyurethane bushings compared to rubber are the following:

- wear resistance;
- elasticity;
- low friction;
- resistance to oils, fuel and petroleum;
- operation at high pressure.

The wall thickness of a polyurethane bushing is determined from the strength condition of thick-walled cylinders loaded with external pressure

$$h = \frac{D}{2}\left(\sqrt{\frac{[\sigma_S]+0.4P_Z}{[\sigma_S]-1.3P_Z}}-1\right), \tag{7}$$

where σ_s – permissible tensile stress for elastomer grade SR55 according to the data of the company «Getzner Werkstof» [3], [11] is taken as 0,061 MPa;

$P_Z = p_1$ – pressure on the walls of the bushing.

The radial deformation of the bushings from uniformly distributed pressure p1 can be determined using the Lame formula [9]:

$$\delta_1 = \frac{r_1 \cdot P_i}{E}\left(\frac{r_2^2+r_1^2}{r_2^2+r_1^2}+\frac{1}{m}\right), \tag{8}$$

where $\frac{1}{m}$ – the ratio of Poisson's ratios of the materials of the bushing and pump body;
r_1 – bushing hole radius;
$r_1 = r_1 + h$ – outer radius of the bushing circle;
E – elastomer elasticity modulus grade SR55.

Rigidity coefficient of polyurethane bushing

$$c_{el} = p_1 \cdot A/\delta_1 = F_0/\delta_1$$

The stiffness coefficient according to the above formula

$$c_{el} = 732{,}5/0{,}00137 = 0{,}535 \cdot 10^6 \text{ N/m}$$

When the elements of the system are connected in series, the reduced rigidity

$$c_r = c_r \cdot c_{el}/(c_r + c_{el}). \tag{9}$$

After substituting the data

$$c_r = 0535 \cdot 66{,}59 \cdot 10^{12}\ (0{,}535 + 66{,}59)10^6 = 0{,}364 \cdot 10^6 N/m$$

Natural frequency of the system

$$\acute{\omega}_a = \sqrt{2c_r/m} = \sqrt{2 \cdot 0{,}364 \cdot\ 10^6/120} = 77{,}8\ s^{-1}$$

The ratio of frequencies of forced and natural oscillations of the system is $308{,}7/77 = 4$, which corresponds to the recommendations for reducing resonance oscillations. Isolation of oscillations by shock absorbers achieves the goal when the ratio of frequencies of forced and natural oscillations, $\acute{\omega}_a/\omega_a > \sqrt{2}$. In practice, the ratio of these frequencies is taken to be 2,5-5. The expression $\left|1/1 - (\omega/\omega_a)^2\right| = |1 - 4^2| = 15$, the transmission coefficient $1/15 =\ 0{,}066$ is included in the recommended range, which characterizes sufficient vibration isolation of the bearings. The force transmitted to the support, taking into account the damping of the system $\mathrm{F} = 2345{,}\cdot 0{,}066 = 15{,}47$ N, i. e. $2345/15{,}47 = 151{,}5$ times less than the force disturbing the oscillations. The reciprocal of the transmission coefficient, expressed in logarithmic units, will determine the vibration isolation (dB) of the shock-absorbing mount.

$$VI =\ 20 \cdot lg\, 1/\mu_t. \tag{10}$$

According to calculations

$$VI =\ 20 \cdot lg\, 1/0{,}066 = 23{,}6\ Hz,$$

which corresponds to the permissible vibration level.

To simulate the vibrations of the 2D630-125 pump body on the foundation, it is necessary to determine the elastic and damping properties of the elements included in this model. To build a foundation for a 2D630-125 pump unit, the class of concrete being laid must be at least 10 (compressive strength in MPa). Concrete of class B25 is accepted, the rigidity of the foundation for the pumping unit according to formula (11)

$$c_f = E_f \cdot A_f/h, \tag{11}$$

where $E_f = 30 \cdot 10^3$ Mpa – modulus of elasticity of concrete under compression;

A_f – the area of the foundation base is taken to be greater than the area of the horizontal projection of the pump;

h – the depth of the underground part of the foundation, for heated rooms the minimum is 0,5 m

$$c_f = 30 \cdot 10^9 \cdot 0{,}33/0{,}5 = 20 \cdot 10^9\ \mathrm{N/m}.$$

Natural frequency of the system

$$\omega_a = \sqrt{20 \cdot 10^9/625} = 5656\ s^{-1}.$$

It is much higher than the frequency of disturbing vibrations, which characterizes the low vibration isolation of the system $\left|1/1 - (\omega/\omega_a)^2\right| = \left|1 - (308{,}7/5656)^2\right|\ = 1$. Transfer coefficient μ_t~1. Therefore, the support reaction is equal to the disturbing force.

For the elastomer material grade SR55, the dynamic modulus of elasticity is $0{,}753 \cdot 10^6$ MPa [12]. Rigidity of the flooring under the pump sole

$$c_{el} = 0{,}753 \cdot 10^6 \cdot\ 0{,}33/0{,}03 = 8{,}74 \cdot 10^6\ \mathrm{N/m}$$

Reduced rigidity of the foundation and elastomer deck

$$c_{r1} = 8{,}74 \cdot 20000 \cdot 10^6/(8{,}74 + 20000) \cdot 10^6 \sim 8{,}74 \cdot 10^6\ \mathrm{N/m}$$

Natural frequency of the system

$$\omega_a = \sqrt{8{,}74 \cdot 10^6/625} = 118{,}25\ s^{-1},$$

below the frequency of disturbing oscillations in $308{,}7/118{,}25 = 2{,}61$, what characterizes sufficient vibration isolation of the system $\left|1/1 - (\omega/\omega_a)^2\right| = \left|1 - (308{,}7/118{,}25)^2\right|\ = 5{,}81$ Load transferred to the support $2345/5{,}81 = 403{,}6$ N.

One of the common methods for assessing the dynamic parameters of mechanical equipment is modelling the movement of the masses of its individual elements [12]. The movement or their kinematic and force interaction are described by systems of differential equations. Mathematical modelling allows us to assess the calculations performed and supplement them with studies that are difficult to conduct without taking into account the dynamics of the process [13]. As shown in Figure 1, the mathematical model of the pump unit consists of two moving masses connected to each other by elastic connections in the form of stiffness coefficients. The mass of the pump casing without the rotor, m_1, and the mass of the rotor, i. e. the shaft with supports and the impeller assembly, m_2. For ease of analysis, the location of the masses and the coordinate of their movement X in Figure 1 and further in the text are assumed to be horizontal, although in reality the oscillations of the system elements occur in the radial, i. e. vertical direction.

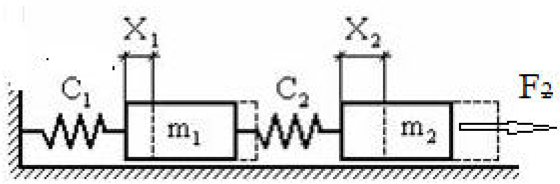

Fig. 1. - Two-mass forced oscillation system

The differential equations of free oscillations of masses have the form:

$$\begin{cases} m_1\ddot{X}_1 + C_1X_1 - C_2(X_2 - X_1) = 0; \\ m_2X_2 + C_2(X_2 - X_1) = 0, \end{cases} \tag{11}$$

frequency equation of a system of two masses:

$$\omega^4 - \left(\frac{c_1 + c_2}{m_1} + \frac{c_2}{m_2}\right)\omega^2 + \frac{c_1c_2}{m_1m_2} = 0 \tag{12}$$

This frequency equation always has two real and positive solutions, i. e. a system with two degrees of freedom has two natural frequencies, which are found from the solution of the quadratic frequency equation [14]:

$$\omega_{1,2} = \frac{0.5(c_1+c_2)}{m_1} + \frac{0.5c_2}{m_2} \mp \sqrt{(0.25\left(\frac{c_1+c_2}{m_1} + \frac{c_2}{m_2}\right)^2 - \frac{c_1c_2}{m_1m_2})} \tag{13}$$

2. Results and discussion

For the pump 2D630-125, the calculation of equations (12) of free oscillations of the system gives the values of natural frequencies $\omega_1 = 749\, s^{-1}$; $\omega_2 = 5666\, s^{-1}$. The frequency of forced oscillations of the pump rotor on rolling bearings at harmonic $k = 3$ is $\omega = 308{,}7 \cdot 3 = 926\, s^{-1}$. It is close to the natural frequency of the system. $\omega_2 = 749\, s^{-1}$, $926/749 = 1{,}23$ frequency ratio, which promotes excitation of near-resonant oscillations of the rotor. The main rotor frequency $\omega = 308{,}7\, s^{-1}$ is below the range of natural frequencies of the bearing support and is not sufficient to excite resonant vibrations. The parameters of the mathematical model presented in Table 1.

Table 1. Parameters of the mathematical model

Parameters	m_1, kg	m_2, kg	c_1, N/m	c_2, N/m	c_{r1}, N/m	c_{r2}, N/m	ω_1, s^{-1}	ω_2, s^{-1}
2D630-125	625	120	$20 \cdot 10^9$	$1331 \cdot 0^6$	$8{,}74\ 10^6$	$364 \cdot 10^3$	0	308,7

After calculating the free oscillations of the system and determining the natural frequencies, the calculation of forced oscillations is carried out.

For an independent study of oscillations caused by one external source, a solution is made to the system of two equations from the second force $F_2(\omega)$, arising from the violation of the shaft alignment. This method has shown its effectiveness and sufficient accuracy in work [15].

When calculating the disturbance from the centrifugal force on the second mass, the first force is taken equal to zero, and the second force $F_2(\omega_2)$, applied to the rotor supports, is calculated under the condition that the disturbance frequency is equal to the rotor frequency ω_2

$$A_1 = F_2c_2/((c_1 + c_2 - m_1\omega_2^2)(c_2 - m_2\omega_2^2) - c_2^2); \tag{14}$$

$$A_2 = F_2(c_1 + c_2 - m_1\omega_2^2)/((c_1+c_2 - m_1\omega_2^2)(c_2 - m_2\omega_2^2) - c_2^2) \tag{15}$$

The calculation of the expression using special programs for calculating algebraic equations on a computer [8] showed (Table 2) that when exciting oscillations with force $F_2(\omega)$ for a 2D630-125 pump without a damping device (DD) at the points where the casing is attached to the foundation, the elastic deformation force of the casing supports (14) on the foundation was $F_{01} = 20 \cdot 10^9 \cdot 1{,}288 \cdot 10^7 = 2576$ N. Calculation of the amplitude of the second mass using expression (15) yielded a value of $A_2 = 0{,}965 \cdot 10^{-5}$ m, the elastic deformation force of the rotor supports without an elastic element was $2F_{12} = 2 \cdot 0{,}965 \cdot 10^{-5} \cdot 133{,}1 \cdot 10^{6} = 2513$ N, (the force per support is 1256 N). The force exciting vibrations for two supports is equal to 2345 N, the transfer of external load to both types of supports occurs approximately equally and this is due to the fact that the transfer coefficient, as shown above, is close in value to one. Calculation of the amplitude of the first mass with an elastic element in the form of a Vibro-damping Elastomeric Plate (VEP) flooring under the pump showed the value $A_1 = -0{,}402 \cdot 10^{-4}$ m, the elastic deformation force of the pump supports with the VEP was $F_{01} = -0{,}402 \cdot 10^{-4} \cdot 8{,}74 \cdot 10^{6} = -351$ N. The deformation of the supports of two masses according to the calculation of the equation taking into account the sign is equal to the difference in amplitudes $A_1 - A_2 = (-0{,}402 + 0{,}247) \cdot 10^{-4} = -0{,}155 \cdot 10^{-4}\ m$, the elastic deformation force of the rotor supports was $2F_{12} = -0{,}155 \cdot 10^{-4} \cdot 1{,}33 \cdot 10^{6} = -2063$ N, (the force per support is 1031,5 N)

Table 2. Elastic deformation forces of supports without elastic elements and with them in pump 2D630-125

Pump 2D630-125			
Disturbing force, N	$F_2(\omega) = 2345\sin308t$		
Force on rotor supports and foundation, N	without DD	VEP on the base	VEP on the foundation and elastic bushings on bearings
$2F_{12}$	2513	2063	18,3
F_{01}	2576	351	27,7
$2F_{12}^{*}$	2564		15,47
F_{01}^{*}	2345	403,6	

Thus, the presence of an elastic support under the base of the pump housing slightly reduces the load from vibration acting on the bearings (from 2513 N to 2063 N). While the reaction on the housing supports to vibration when using VEP is significantly reduced. While the reaction on the housing supports from vibration when using VEP is significantly reduced from 2063 N to 351 N, i. e. by 5.87 times, the use of elastic elements made of elastomer as a flooring for the pump housing and the installation of dampers made of polyurethane bushings under the outer ring of the rolling bearing reduce the vibration loads perceived by the foundation from 2576 N to 27.7 N, and on rolling bearings from 2513 N to 18.3 N.

Table 2 shows the elastic deformation forces of the supports F_{01}, F_{12} are presented based on the results of mathematical modelling and calculation using the transfer coefficient (shown in the table with an asterisk). The results are almost identical with a small error, which confirms the objectivity of the adopted methods and the reliability of the calculation results.

Conclusions

The following conclusions follow from the analysis of the results:

- the amplitude of the load oscillations depends on the ratio of the frequencies of forced and natural oscillations. Due to the high frequency of natural oscillations of the bearings and lower frequencies of oscillations of the centrifugal force, the efficiency of vibration isolation is reduced.
- it is shown that the use of elastic elements made of elastomer as a flooring for the pump casing slightly reduces the vibration loads acting on the rotor supports when oscillations are excited by the centrifugal force of the rotor. While the reaction on the body supports from vibration when using the VEP is significantly reduced
- it is shown that the use of elastic elements made of elastomer as a flooring for the pump casing and the installation of dampers made of polyurethane bushings under the outer ring of the rolling bearing reduce the vibration loads perceived by the foundation and bearings.

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