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# Free and Forced Vibrations of the Carrier Beam of the Vehicle Chassis

**Akhmediev S.K., Filippova T.S., Oryntayeva G.Zh.,**
**Tazhenova G.D., Mikhailov V.F.***
Abylkas Saginov Karaganda Technical University, Karaganda, Kazakhstan
*corresponding author

**Abstract.** The article is dealing with the stress-strain state of the carrier beam of the two-axle vehicle chassis for a dynamic perturbing load that occurs in the process of driving along the unevenness of the road, taking into account the elastic characteristics of the spring suspension. When analyzing the ride smoothness, it is necessary to take into account low-frequency forced vibrations caused by road irregularities, as well as free low-frequency vibrations. The dynamic calculation was carried out by the force method. In this case, the expansion of a given arbitrary dynamic load is performed in terms of the main modes of vibration. The calculation for free and forced vibrations was made by an exact analytical method, taking into account the elastic pliability of the axial supports of the beam. In the course of the study, the external dynamic forces acting on the structure under consideration, reduced to three-point masses, were determined and three forms of free vibrations were obtained. The results of the study can be used to optimize the design of the truck chassis and to improve their reliability and safety in operation. By understanding the behavior of a suspension system under dynamic loads, engineers can design stronger, more reliable designs that can withstand the stress of transporting heavy loads over rough roads. In general, this study contributes to the development of more efficient and safer transport systems.

**Keywords:** free and forced vibrations; chassis; carrier beam; stress-strain state; amplitude.

## Introduction

Any complex multi-mass mechanical systems, including vehicles interact with the external environment. This system includes sprung masses such as a body, a driver, passengers, a cargo and unsprung masses of axles. The masses interact with each other through the elastic and dissipative elements of the suspension, tires and seats that allow their moving relative to each other.

There are a lot of mathematical models that are used to study the dynamics of a vehicle. Works [1]–[4] present a complete analysis of various models from the designer's point of view, for example, changing the suspension stiffness, the damping coefficient, optimizing the form of its elements using the modal finite element analysis [5]. In [6], a nonlinear model of a dynamic system was considered using the Simscape environment in the MATLAB/Simulink. The results show a significant reduction in the amount of displacement and acceleration of the sprung mass during rebound and roll compared to the linear model, therefore, the non-linear spring elements work well in most static and dynamic conditions.

When analyzing ride smoothness, it is necessary to take into account low-frequency forced vibrations caused by road irregularities, as well as free low-frequency vibrations. For trucks, it is sufficient to take into account vibrations of the body and suspension, in which vertical vibrations of the masses and angular vibrations of the body in the longitudinal and transverse vertical planes of the vehicle are analyzed. The quality of the suspension depends directly on the amplitude-frequency characteristics of the kinematic and vibration effects [7], [8]. In [9] it was found that increasing nonlinear stiffness leads to changing the magnitude of oscillation amplitudes and frequencies that affect the overall stability of the vehicle. In [10], forced vibrations of a four-axle vehicle with a double spring suspension were studied. For its mathematical model a numerical solution was obtained for the critical value of external excitation, within which vibration becomes stable. In this case, the suspension parameters were selected from the required amplitude value.

The vehicle movement occurs in conditions of uneven roads, which leads to vibrations of the body, seats and is accompanied by vibration loads on the human body and the vehicle mechanisms. In [11], the influence of road roughness, vehicle speed, suspension stiffness and damping on vehicle characteristics was studied. In addition, inverse problems were solved, such as determining the reaction of the road surface when the vehicle was moving [12].

One of the ways to reduce vibration loads is to regulate the damping of oscillations of the sprung masses of the vehicle suspension system. In works [13], [14], the dynamics of the vehicle suspension behavior when it moves along the unevenness of the roadway, as well as through the forest area, is studied. The results show that the proposed suspension can provide a significant reduction in vibration levels, and the appropriate selection of the material leads to decreasing the deflection and resulting stresses, improving dynamic performance and increasing the service life.

The developers of a new technology pay great attention to the development of new suspension elements in the form of inerter [15] and hydraulic integrated suspension [16].

**1. Methods and solutions**

When designing vehicles of various designs and carrying capacity, it is necessary to calculate the load-bearing structures of the vehicle chassis, taking into account their movement along the unevenness of the road, i.e., to calculate them under kinematic disturbances.

Forced vibrations of chassis structures are caused by kinematic perturbation of the spring suspension of a moving vehicle.

The object of study is the carrier beam of the vehicle chassis with constant bending stiffness with resiliently pliable supports (A, B) (Figure 1) that work in bending from the dynamic impact caused by the kinematic movement of the spring suspension.

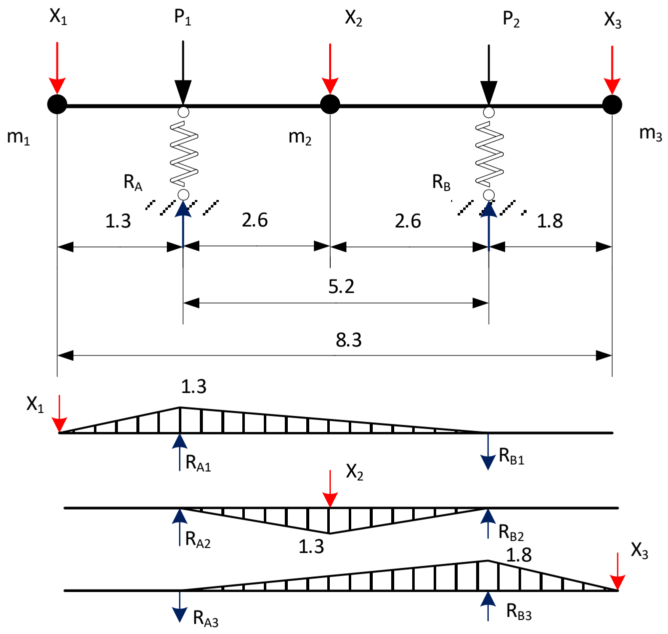


**Fig. 1.** - The carrier beam of the two-axle vehicle chassis

Dynamic calculations of the above construction will be carried out by the force method. Direct dynamic calculation of the structure under study is carried out in the following algorithmic sequence:

1. In order to simplify the calculation, there is first performed the expansion of the given arbitrary dynamic load in terms of the main vibration modes.

2. There is performed the calculation for free vibrations (in the absence of a disturbing force) as a system with three degrees of freedom in order to determine the frequency spectrum and modes of natural vibrations with studying the dependence of frequency characteristics on the changes in the degree of pliability (cushioning) of the chassis supports.

3. There is performed the calculation for forced vibrations from the disturbing harmonic load that occurs when vehicles move along uneven roadways with studying the changes in the amplitudes of dynamic movements from the degree of pliability of the chassis wheel bearings.

**2. Calculation for free vibrations**

Let's calculate the coefficients of the pliability matrix (Figure 1). Vereshchagin's method, representative by graph multiplication method, shows the effective ability to determine displacement resulting from deformation body of bending structures. The advantages of this method are the integrating process derived from Maxwell-Mohr:

$$\begin{aligned}
\delta_{11} &= \int_0^l \frac{(M_1^2)}{EJ_x}dx + \frac{\sum_{i=1,2}(R_{1,i}^2)}{C_i} = 2.375 \cdot 10^{-3}\, kN \cdot m \\
\delta_{22} &= \int_0^l \frac{(M_2^2)}{EJ_x}dx + \frac{\sum_{i=1,2}(R_{2,i}^2)}{C_i} = 1.2054 \cdot 10^{-3} kN \cdot m \\
\delta_{33} &= \int_0^l \frac{(M_3^2)}{EJ_x}dx + \frac{\sum_{i=1,2}(R_{3,i}^2)}{C_i} = 3.402 \cdot 10^{-3} kN \cdot m \\
\delta_{12} = \delta_{21} &= \int_0^l \frac{(M_1)(M_2)}{EJ_x}dx + \sum_{i=1,2} \frac{(R_{1,i} \cdot R_{2,i})}{C_i} == -0.144 \cdot 10^{-3}\, kN \cdot m
\end{aligned} \tag{1}$$

$$\delta_{13} = \delta_{31} = \int_0^l \frac{(M_1)(M_3)}{EJ_x} dx + \sum_{i=1,2} \frac{(R_{1,i} \cdot R_{3,i})}{C_i} == -0.0098 \cdot 10^{-3}\ kN \cdot m$$

$$\delta_{32} = \delta_{23} = \int_0^l \frac{(M_2)(M_3)}{EJ_x} dx + \sum_{i=1,2} \frac{(R_{2,i} \cdot R_{3,i})}{C_i} == -0.365 \cdot 10^{-3}\ kN \cdot m$$

where $\delta_{ii}$ is the coefficient of the pliability matrix, $M_i$ is the bending moment, $C_i$ is the coefficient of stiffness, $E$ is the elastic modulus, $J_x$ is the moment of inertia, $R_{i,i}$ is the support reaction.

Let's make a secular equation ($\text{т} = 0.25 \cdot 10^4\ kg$, $\text{C} = 1{,}15 \cdot 10^4 \frac{kN}{m}$):

$$D = m \begin{vmatrix} \delta_{11} - \lambda & \delta_{12} & \delta_{13} \\ \delta_{21} & \delta_{22} - \lambda & \delta_{23} \\ \delta_{31} & \delta_{32} & \delta_{33} - \lambda \end{vmatrix} = 0 \tag{2}$$

where $D$ is the matrix determinant, $\lambda_i$ is the root of the secular equation, $m$ is the mass.

$$\lambda_i = \frac{1}{\omega_i^2 m} \qquad \omega_i = \sqrt{\frac{1}{\lambda_i m}} \tag{3}$$

where $\omega_i$ is the circular frequence.

Solution (2) gives the following values:

$$\lambda_1 = 8{,}653 \cdot 10^{-4} \frac{s^2}{kg} \quad \omega_3 = 59{,}52\ s^{-1}$$

$$\lambda_2 = 5{,}979 \cdot 10^{-4} \frac{s^2}{kg} \quad \omega_2 = 40{,}90\ s^{-1}$$

$$\lambda_3 = 2{,}823 \cdot 10^{-4} \frac{s^2}{kg} \quad \omega_1 = 34\ s^{-1}$$

Based on the intrinsic vectors $\bar{v}_{ik}$ ($i = 1, 2, 3;\quad k = 1, 2, 3$) there is built the forms of intrinsic vibrations in the form of the "standing" waves (Figure 2).

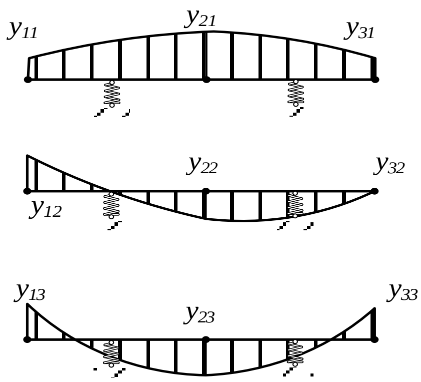


**Fig. 2.** - Beam intrinsic vibrations forms

**II Calculation for forced vibrations**

Let's consider the case of uniform motion of a vehicle along the road with the speed $v$ [17]–[19].

In this case, the abscissa of motion is $x = vt$ and the path profile has the equation $x = \Delta(vt)$. Then the perturbing inertia force is taken in the form of a harmonic load

$$P_i(t) = P_{0,i} \sin \theta t \qquad (i = 1, 2) \tag{4}$$

where $P_i(t)$ is the perturbing force, $P_{0,i}$ is the amplitude perturbing force.

$$\theta = \pi v / l_0 \tag{5}$$

where $\theta$ is the disturbing force frequency, $v$ is the velocity of movement, $l_0$ is the average length.
The amplitude perturbing force is

$$P_{0,i} = m_i f_{0,i} \left(\frac{\pi^2 v^2}{l_0^2}\right) = m_i f_{0,i}(\theta^2). \tag{6}$$

where $f_{0,i}$ is the kinematic excitation parameter.

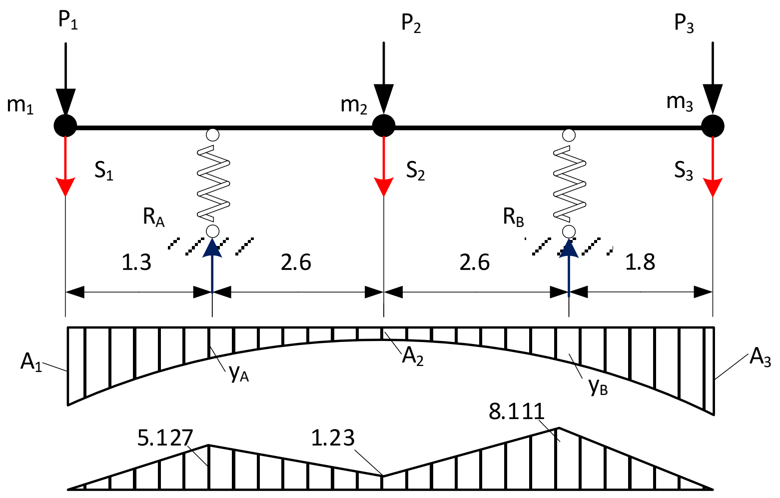


**Fig. 3.** - Forced vibration scheme

Let's consider the steady-state oscillations of the system (in the absence of movement resistance forces). The parameters of forced vibrations are as follows:

a) at an arbitrary moment

- $y_i(t) = A_i \sin \theta t$ is the displacement, $A_i$ is the displacement amplitude;
- $J_i(t) = -m_i \ddot{z}_i = \theta^2 m_i A_i \sin \theta t$ is the inertia force;
- $P_i(t) = P_{0i} \sin \theta t$ is the perturbing force;
- $S_i(t) = (P_{0i} + \theta^2 m_i A_i) \sin \theta t$ is the external dynamic force;

b) in the amplitude state

- $A_i$;
- $J_i = \theta^2 m_i A_i$;
- $P_{0i}$;
- $S_i = P_{0i} + J_i = P_{0i} + \theta^2 m_i A_i$.

For the further calculations there is accepted the following:
$v = 16.67\ \frac{m}{s}$; $l_0 = \frac{l}{4} = 2.075\ m$; ($l = 8.3\ m$, Figure 1).

Based on formula (5):

$$\theta = \frac{3{,}14 \cdot 16{,}67}{2{,}075} = 25{,}23\ s^{-1}.$$

The pliability matrix of the system (with $n = 3$):

$$[\delta] = \begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{21} & \delta_{22} & \delta_{23} \\ \delta_{31} & \delta_{32} & \delta_{33} \end{bmatrix} \tag{7}$$

Having accepted $C_1 = 1{,}15 \cdot 10^4\ \frac{kN}{m}$, there is obtained:

$$[\delta] = 10^{-3} \begin{bmatrix} 2.375 & -0.144 & -0.0098 \\ -0.144 & 1.205 & -0.365 \\ -0.0098 & -0.365 & 3.402 \end{bmatrix}$$

The deflection vector, taking into account (7):

$$\vec{A} = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = [\delta]\vec{S} = \\ = 10^{-3} \begin{bmatrix} 2.375 & -0.144 & -0.0098 \\ -0.144 & 1.205 & -0.365 \\ -0.0098 & -0.365 & 3.402 \end{bmatrix} \cdot \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} \quad (8)$$

$$\vec{\Delta}_{i,P_0} = \begin{bmatrix} \Delta_{1P_0} \\ \Delta_{2P_0} \\ \Delta_{3P_0} \end{bmatrix} = [\delta]\vec{P}_{0,i} = \\ = 10^{-3} \begin{bmatrix} 2.375 & -0.144 & -0.0098 \\ -0.144 & 1.205 & -0.365 \\ -0.0098 & -0.365 & 3.402 \end{bmatrix} \cdot \begin{bmatrix} P_{01} \\ P_{02} \\ P_{03} \end{bmatrix} \quad (9)$$

By (6) we obtained:

$$P_{0,i} = m_i f_{0,i}(\theta^2) = 0{,}25 f_{0,i}(25{,}23)^2 = 159{,}14 f_{0,i}$$

The amplitude equations [20]:

$$\begin{cases} \left[\delta_{11}m_1 - \left(\frac{1}{\theta^2}\right)\right] A_1 + \delta_{12}m_2A_2 + \delta_{13}m_3A_3 = -\frac{\Delta_{1P_0}}{\theta^2} \\ \delta_{21}m_1A_1 + \left[\delta_{22}m_2 - \left(\frac{1}{\theta^2}\right)\right] A_2 + \delta_{23}m_3A_3 = -\frac{\Delta_{2P_0}}{\theta^2} \\ \delta_{31}m_1A_1 + \delta_{32}m_2A_2 + \left[\delta_{33}m_3 - \left(\frac{1}{\theta^2}\right)\right] A_3 = -\frac{\Delta_{3P_0}}{\theta^2} \end{cases} \quad (10)$$

The equations for amplitudes of external dynamic forces:

$$\begin{cases} \left[\delta_{11} - \left(\frac{1}{m_1\theta^2}\right)\right] S_1 + \delta_{12}S_2 + \delta_{13}S_3 = -\frac{P_{01}}{m_1\theta^2} \\ \delta_{21}S_1 + \left[\delta_{22} - \left(\frac{1}{m_2\theta^2}\right)\right] S_2 + \delta_{23}S_3 = -\frac{P_{02}}{m_2\theta^2} \\ \delta_{31}S_1 + \delta_{32}S_2 + \left[\delta_{33} - \left(\frac{1}{m_3\theta^2}\right)\right] S_3 = -\frac{P_{03}}{m_3\theta^2} \end{cases} \quad (11)$$

Here

$$m_1 = m_2 = m_3 = 0{,}25 \cdot 10^4 \ kg; \ \theta = 25{,}23 \ s^{-1}.$$

Let's open equation (9) taking into account the initial data (with $\theta/\omega_1 = 0{,}742$):

$$\begin{cases} -3.909S_1 - 0.144S_2 - 0.0098S_3 = -15.899 \sin 25.23t \\ -0.144S_1 - 5.079S_2 - 0.365S_3 = -17.72 \sin 25.23t \\ -0.0098S_1 - 0.365S_2 - 2.882S_3 = -14.139 \sin 25.23t \end{cases} \quad (12)$$

Having solved system (12), there is obtained:

$$S_1 = 3.944 \cdot 10^4 N, S_2 = 3.053 \cdot 10^4 N, S_3 = 4.506 \cdot 10^4 N.$$

Figure 3 shows the diagram of bending moments under external dynamic forces. Based on (8), there is determined

$$A_1 = 8.765 \cdot 10^{-3} \ m, \ A_2 = 1.917 \cdot 10^{-3} \ m, \ A_3 = 14.176 \cdot 10^{-3} \ m.$$

Based on the beam deflection curve (Figure 3) there is expressed the beam deflections dependence through the interpolation polynomial [21] with ($n = 2$ is the degree of the polynomial, $a_0 = A_1$):

$$y(x) = a_0 + a_1x_i + a_2x_i^2 = 10^{-3}(8.765 + a_1x_i + a_2x_i^2) \quad (13)$$

According (13) (Figure 3) there is written down the system of algebraic equations for determining the values of $a_1$, $a_2$, ($x_1 = 3.9m$, $x_2 = 8.3m$):

$$\left.\begin{aligned} 8.765\cdot 10^{-3}+3.9a_1+15.21a_2=1.917\cdot 10^{-3}\\ 8.765\cdot 10^{-3}+8.3a_1+68.89a_2=14.176\cdot 10^{-3}\end{aligned}\right\} \quad (14)$$

Solving system (14), there is determined:

$$a_1=-3.89\cdot 10^{-3};\quad a_2=0.547\cdot 10^{-3} \quad (15)$$

Substitute expressions (15) into equation (13):

$$y(x_i)=(8.765-3.89x_i+0.547x_i^2)\cdot 10^{-3} \quad (16)$$

where $y(x_i)$ is the ordinate of displacement and the interpolation polynomial of 2nd degree obtained (16).
Based on (16) there is calculated:

- $x_i=1.3\ m$ (on the A support): $y_A=4.6337\cdot 10^{-3}\ m$;
- $x_i=6.5\ m$ (on the B support): $y_B=6.597\cdot 10^{-3}\ m$.

The accuracy of calculating the intermediate deflections of a beam can be increased by taking into account the interpolation polynomial of a higher degree, i.e., taking $n>2$.

Next, there will be studied the effect of the elastic pliability of supports *A* and *B* on the dynamic forces $P_1$, $P_2$ of the beam in Figure 1. Let's take the values of the pliability coefficients of supports *A*, *B* in the range $(0 \ldots 0.87\cdot 10^{-4}\ \frac{m}{kN})$ (the results are in Table 1).

According to Table 1, there are constructed graphs of the circular frequencies of free vibrations dependence on the pliability coefficients of supports *A* and *B* (indicated by $k_i$ ).

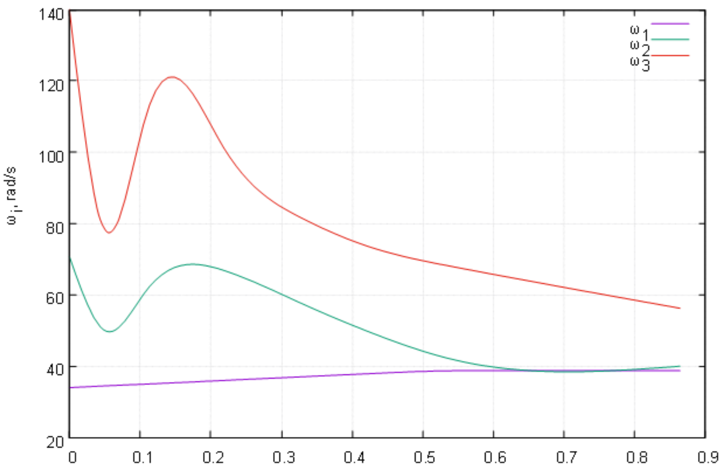


**Fig. 4.** - The $\omega_i=f(k_i)$ $(k_i\cdot 10^{-4}\frac{m}{kN})$ dependences

Let's write the expressions of interpolation polynomials according to Table 1 for the analytical determination of the circular frequencies of free vibrations $\omega_i$ $(i=1,\ 2,\ 3)$ and the amplitudes of dynamic displacements $y_i$ $(i=1,\ 2,\ 3)$ depending on changing the $C_i$ and $k_i$ values.

**Table 1.** Dynamic parameters for the beam depending on the coefficients of stiffness $C_i$ and pliability ($k_i=1/C_i$) of the *A* and *B* supports

| $C_i\cdot 10^4$, $\frac{kN}{m}$ | $k_i\cdot 10^{-4}$, $\frac{m}{kN}$ | Circular frequencies $\omega_i, s^{-1}$ | | | Dynamic movement amplitudes, m | | |
|---|---|---|---|---|---|---|---|
| | | $\omega_1$ | $\omega_2$ | $\omega_3$ | $A_1\cdot 10^{-3}$ | $A_2\cdot 10^{-3}$ | $A_3\cdot 10^{-3}$ |
| 1.15 | 0.87 | 34.0 | 40.90 | 59.52 | 8.765 | 1.917 | 14.176 |
| 2.3 | 0.434 | 36.67 | 49.24 | 74.16 | 5.286 | -0.894 | 9.5723 |
| 10 | 0.1 | 38.74 | 64.42 | 109.69 | 3.6988 | -2.00 | 6.072 |
| 20 | 0.05 | 35.83 | 51.57 | 83.28 | 7.005 | 0.595 | 11.054 |
| 30 | 0.033 | 35.95 | 48.27 | 75.95 | 7.565 | 1.01 | 12.04 |
| ∞ | 0.0 | 38.48 | 70.48 | 135.96 | 2.561 | 7.89 | 5.36 |

According to Table 1, it can be seen that with increasing the pliability of supports *A*, *B* (Figure 3), the circular frequencies of natural vibrations of the beam first increase monotonically, and then decrease monotonically, while the values reach the minimum $\omega_i$ $(i=1,\ 2,\ 3)$ at the value of $k_i\approx 0{,}21\cdot 10^{-4}\ \frac{m}{kN}$.

The dependences of circular frequencies (by analogy with (16)) are as follows:

$$\left.\begin{aligned} \omega_1 &= 34.0 + 40.63k_i - 79.182k_i^2 \\ \omega_2 &= 40.90 + 238.53k_i - 503.16k_i^2 \\ \omega_3 &= 59.52 + 532.49k_i - 1144.86k_i^2 \end{aligned}\right\}. \quad (17)$$

The first derivatives from the frequencies:

$$\begin{aligned} \omega_1' &= 40.63 - 79.182k_i = 0;\ k_i^1 = 0.513 \cdot 10^{-4}\ \frac{m}{kN}, \\ \omega_2' &= 238.53 - 503.16k_i = 0;\ k_i^2 = 0.474 \cdot 10^{-4}\ \frac{m}{kN}, \\ \omega_3' &= 532.49 - 1144.86k_i = 0;\ k_i^3 = 0.465 \cdot 10^{-4}\ \frac{m}{kN}. \end{aligned} \quad (18)$$

Now let's study the dynamic movement amplitudes $A_i$ depending on the changing of pliability coefficients $k_i$.

We accept the next values $C_2 = 2.3 \cdot 10^4\ \frac{kN}{m}$, $\theta = 25.23\ s^{-1}$, т $= 0.25 \cdot 10^4\ kg$. According to (7) and (11) we obtain:

$$[\delta] = 10^{-3}\begin{bmatrix} 1.6685 & -0.3608 & 0.2619 \\ -0.3608 & 0.9883 & -0.5826 \\ 0.2619 & -0.5826 & 2.6955 \end{bmatrix}$$

$$\begin{cases} -4.6155S_1 - 0.3608S_2 + 0.2619S_3 = -15.899 \\ -0.3608S_1 - 5.296S_2 - 0.5826S_3 = -17.72 \\ 0.2619S_1 - 0.5826S_2 - 3.5885S_3 = -14.139 \end{cases}$$

Having solved system, there is determined:

$$S_1 = 3.457\ \cdot 10^4 N;\ \ S_2 = 2.582\ \cdot 10^4 N;\ \ S_3 = 3.7734\ \cdot 10^4 N.$$

$A_1 = 5.286 \cdot 10^{-3}\ m,\ A_2 = -0.8939 \cdot 10^{-3}\ m,\ A_3 = 9.5723 \cdot 10^{-3}\ m$ .

For $C_3 = 10 \cdot 10^4\ kN/m$, $\theta = 25.23\ s^{-1}$, т $= 0.25 \cdot 10^3 kg\ s^2/m$:

$$[\delta] = 10^{-3}\begin{bmatrix} 1.125 & -0.5282 & 0.4711 \\ -0.5282 & 0.8209 & -0.75 \\ 0.4711 & -0.75 & 2.005 \end{bmatrix}$$

$$\begin{cases} -5.159S_1 - 0.5282S_2 + 0.4711S_3 = -15.899 \\ -0.5282S_1 - 5.4631S_2 - 0.75S_3 = -17.72 \\ 0.4711S_1 - 0.75S_2 - 4.279S_3 = -14.139 \end{cases}$$

There is obtained:

$$S_1 = 3.1187\ \cdot 10^4\ N;\ \ S_2 = 2.502\ \cdot 10^4\ N;\ \ S_3 = 3.2092\ \cdot 10^4\ N.$$

$$A_1 = 3.6988 \cdot 10^{-3}\ m,\ A_2 = -2.0 \cdot 10^{-3}\ m, \qquad A_3 = 6.0272 \cdot 10^{-3}\ m$$

Let's write down interpolation polynomials (11) for dynamic movement amplitudes $A_i$ (based on the data of Table 1):

a) for $A_1$:

$A_1 = a_0 + a_1 x_i + a_2 x_i^2;\ a_0 = 8.765$

$$\begin{cases} \Delta x_1 = 0.87 - 0.434 = 0.436 \\ \Delta x_2 = 0.1 - 0.434 = 0.334 \end{cases}$$

$5.286 = 8.765 + a_1(0.436) + a_2(0.436)^2$

$3.6988 = 8.765 + a_1(0.334) + a_2(0.334)^2$

$$\begin{cases} 0.436a_1 + 0.190a_2 = -3.479; \\ 0.334a_1 + 0.11156a_2 = -5.0662 \end{cases}$$

$$\begin{vmatrix} a_1 = -38.723 \\ a_2 = 70.55 \end{vmatrix}$$

$$A_1 = 10^{-3}(8.765 - 38.723k_i + 70.55k_i^2) \quad (19)$$

b) for $A_2$:

$$A_2 = a_0 + a_1 x_i + a_2 x_i^2; \;\; a_0 = 1.917$$
$$-0{,}894 = 1.917 + a_1(0.436) + a_2(0.436)^2$$
$$-2.00 = 1.917 + a_1(0.334) + a_2(0.334)^2$$
$$\begin{cases} 0.436a_1 + 0.190a_2 = -2.811 \\ 0.334a_1 + 0.11156a_2 = -3.917 \end{cases}$$
$$\begin{vmatrix} a_1 = -29.03 \\ a_2 = 51.825 \end{vmatrix}$$
$$A_2 = 10^{-3}(1.917 - 29.03k_i + 51.82k_i^2) \quad (20)$$

c) for $A_3$:

$$A_3 = a_0 + a_1 x_i + a_2 x_i^2; \;\; a_0 = 14{,}176$$
$$9.5723 = 14.176 + a_1(0.436) + a_2(0.436)^2$$
$$6.072 = 14.176 + a_1(0.334) + a_2(0.334)^2$$
$$\begin{cases} 0.436a_1 + 0.190a_2 = -4.6037 \\ 0.334a_1 + 0.11156a_2 = -8.104 \end{cases}$$
$$\begin{vmatrix} a_1 = -69.173 \\ a_2 = 134.505 \end{vmatrix}$$
$$A_3 = 10^{-3}(14.176 - 69.173k_i + 134.505k_i^2) \quad (21)$$

According to equations (19-21), Table 1 is supplemented (with $k_i = 0.05$; $k_i = 0.033$).
Figure 5, according to Table 1, shows the dependence of the amplitudes of the dynamic displacements $A_i$ ($i = 1, 2, 3$) of the beam (Figure 3) depending on the pliability coefficients of the supports ($A$, $B$) (on the values $k_i$).

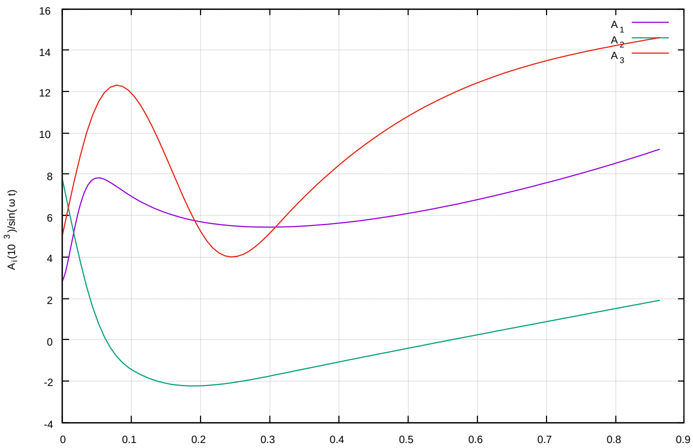


**Fig. 5.** - The $A_i = f(k_i)$ $(k_i \cdot 10^{-4}\frac{m}{kN})$ dependences

## Conclusions

In this work, there is studied the stress-strain state (SSS) of a carrier beam for the two-axle truck chassis for the dynamic perturbing load that occurs during the movement of the vehicle (the calculation for kinematic excitation $x = \Delta(t)$) along the road irregularities, taking into account the elastic characteristics of the spring suspension.

In the process of studies, the external dynamic forces acting on the structure under consideration were determined, reduced to three-point masses. The dependencies $\omega_i = f(k_i)$ , $A_i = f(k_i)$ were given.

For the accepted coefficient of compliance of supports $A$ and $B$ (the $k = 0.87 \cdot 10^{-4}\ \frac{m}{kN}$ value) there were determined:

- the forms of free vibrations at frequencies $\omega_1, \omega_2, \omega_3$;
- the diagram of amplitudes of dynamic changes;
- the diagram of amplitudes of dynamic bending moments.

According to the results of the study, it was established: all three forms of free vibrations are complex, they have one, two or three "standing" waves with alternating plus and minus signs with zero displacements at the locations of the corresponding masses $\text{т}_1, \text{т}_2, \text{т}_3$.

Here there is observed the effect of the elastic pliability of the supports A, B.

When changing the value of the pliability coefficient $k_i$ (Table 1), the following is observed:

- the main tone $\omega_1$ is almost independent of the value $k_i$ (Figure 4);
- the first ($\omega_2$) and second ($\omega_3$) overtones change along a complex trajectory: they either sharply decrease, or sharply increase (at $k_i = 0, 0 \dots 0.324 \cdot 10^{-4}\ m/kN$), then stabilize (monotonically decreasing at $k_i > 0.216 \cdot 10^{-4}\ m/kN$);
- in Figure 5, the nature of changing the amplitudes of dynamic mass displacements $A_i$ ($i = 1,\ 2,\ 3$) is also quite complex: the amplitude of the second mass ($A_2$) (in the span of the beam) changes monotonously, while the amplitudes of the first ($A_1$) and third ($A_3$) masses (on the beam consoles) have a complex-changing character with a sharp increase and decrease within the $k_i = 0, 0 \dots 0.216 \cdot 10^{-4}\ m/kN$ limits and a monotonous change in values at $k_i > 0.216 \cdot 10^{-4}\ m/kN$.

The proposed theoretical developments and applied results can be used in scientific research in the field of mechanics of a solid deformable body, as well as in the process of designing load-bearing structures of various types in mechanical engineering, construction and vehicles, i.e. when solving specific problems of strength, rigidity and stability that arise in the process of design and construction.

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**Information of the authors**

**Akhmediev Serik Kabultaevich**, candidate of technical sciences, Abylkas Saginov Karaganda Technical University
e-mail: s.ahmediev@kstu.kz

**Filippova Tatyana Silinyevna**, candidate of technical sciences, Abylkas Saginov Karaganda Technical University
e-mail: tsxfilippova@mail.ru

**Oryntayeva Gulzhaukhar Zhunuskhanovna**, senior lecturer, Abylkas Saginov Karaganda Technical University
e-mail: oryntaeva70@mail.ru

**Tazhenova Gulzada Dauletkhanovna**, candidate of technical sciences, Abylkas Saginov Karaganda Technical University
e-mail: gulzada_2604@mail.ru

**Mikhailov Valentin Feliksovich**, candidate of technical sciences, Abylkas Saginov Karaganda Technical University
e-mail: v.mihaylov@kstu.kz